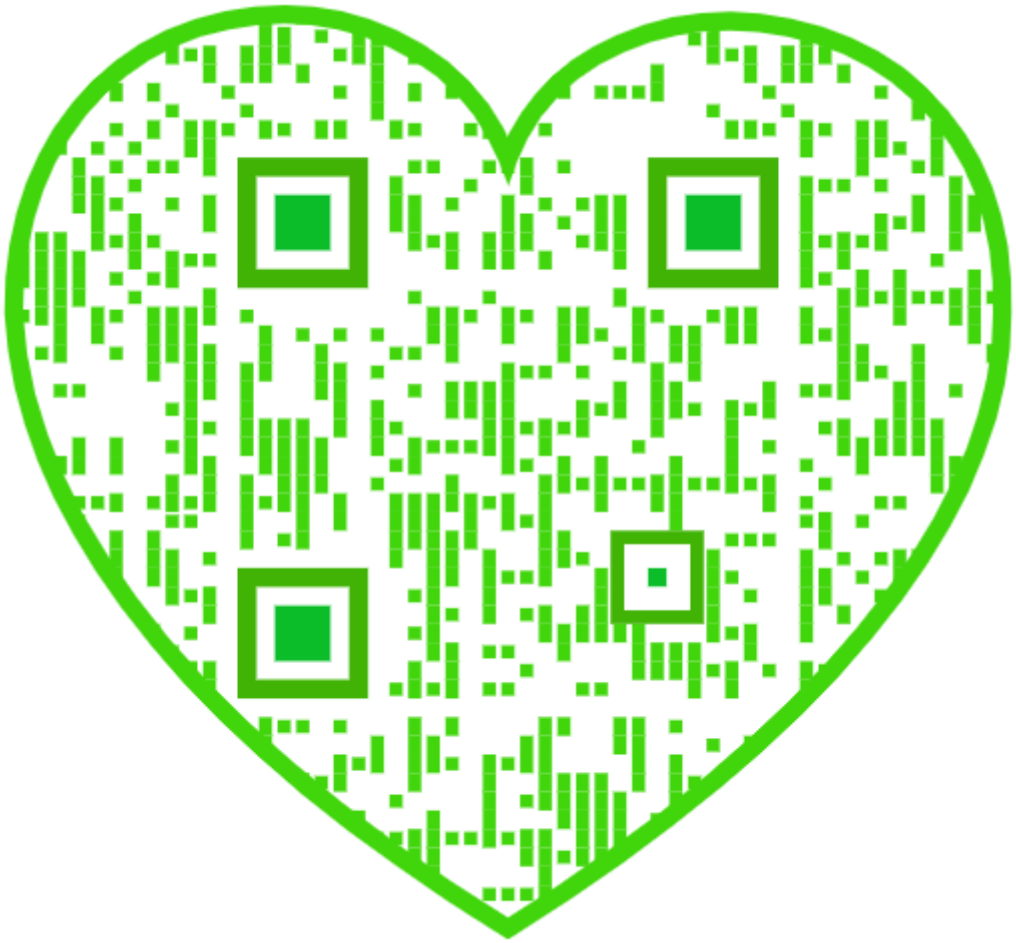

LIPPINCOTT'S

ELEMENTARY ARITHMETIC

EMBRACING

THE SCIENCE AND PRACTICAL APPLICATIONS OF NUMBERS

BY

J. MORGAN RAWLINS, A.M.

AUTHOR OF "LIPPINCOTT'S PRACTICAL ARITHMETIC" AND "LIPPINCOTT'S MENTAL ARITHMETIC"

PHILADELPHIA

J. B. LIPPINCOTT COMPANY

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UNIV. OF CALIFORNIA

L. M. Tarnham

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PREFACE.

"In all the affairs of life, the arithmetical part of the business is the dominant one." How many and how much have we? How many and how much do we want? are questions that constantly obtrude themselves for answer. "Arithmetic is the conclusive science that men have to apply, all their days, to all their affairs." Of all the sciences, therefore, with which men have to do, none deserves more intelligent study or more prudent application than the arithmetical one. Memory alone cannot deal with it or comprehend it. Pupils must be trained *to see, to hear, to think*, in order to grasp its truths and learn to apply them. We dare say that the philosophical study of no other subject will impart to the mind of youth a higher degree of acuteness and penetration. "It makes men subtile," said Lord Bacon.

Very few pupils, we presume, enter school for the first time who have not some idea of number and of numerical combinations; but this knowledge, incidentally gathered here and there, necessarily lies in the mind in no definite or connected order. It is important, therefore, that when pupils begin the systematic study of Arithmetic, they begin with the very first lesson, so that what they already know may be set in order and be made the basis of what is next to be learned. Beginners should be

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led without delay to perceive that the lesson learned to-day is little more than the cultivated ground out of which is to grow the lesson of to-morrow.

It is a fatal error, only too common, to start a child to study where that which he is asked to learn is out of touch with that which he already knows. Pupils should be taught very early to keep an accurate separation of the known from the unknown, and "to be careful not to stamp a thing as known" until they have fully mastered it in all its relations to that which they know, and have done so "in that way which conscience calls *honest.*"

The following Elementary Treatise on Arithmetic has been prepared with the view of presenting to both teacher and pupil a thoroughly systematic and gently graded scheme in which they may together make daily progress in scientific knowledge of the subject, and by a mutual interest in the work gather by diligence many of the best fruits of industry.

Nothing, from beginning to end, has been written as mere verbiage, undeserving of attention. Every word has a measure of significance, and every sentence in the book is there for the single purpose of being understood.

J. M. R.

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GENERAL SUGGESTIONS.

PART I. of this book presupposes the occasional necessity of introducing pupils to the first rudiments of the subject.

In developing the idea of number, and teaching the simple numerical operations, real objects are greatly to be preferred to pictures, however artistic and striking they may be. But what is to be preferred to all other agencies, whether pictures or real objects, or even the book itself, is the voice and action of the live and intelligent teacher, without which little of educational value is ever accomplished in school. Object lessons, however, are of great value in illustrating and impressing the teacher's meaning; but no objects should be used except those of simple form and construction, lest the mind of the pupil be diverted by them from the primary object in view, namely, the inculcation of the idea of number and of numerical combinations.

When objects are employed, the youthful pupil should be allowed to take them in his own hands and give proof of knowledge gained by showing without help how and what they explain. This will be to his liking, and liking is a supreme element of learning.

The successful teacher is not he who does both his own and the pupil's work, but he who best directs the pupil's

activities, leads him to love learning, and to overcome difficulties by his own efforts.

In reciting, pupils should be required to give their answers in complete sentences, and that, too, without hesitation or counting. They should be able to say "4 and 3 are 7," or "4 less 3 equals 1," as promptly and with as little apparent effort as they would spell a word of three letters. In sight exercises, where rapidity is the object sought, results alone should be given; thus: "seven," "one."

The use of concert exercises should not be constant, but only occasional, as a ready means to give diversity or to revive flagging interest. As far as possible, the members of a class should be drilled individually, for each has a separate and distinct individuality that demands and must receive from the teacher the carefulest recognition. Every exercise should be made so interesting as to engage the undivided attention of every pupil, and to effect this they must of necessity be lively, varied, and above all *brief*. The very best judgment of the teacher is demanded here. The processes of Addition and Subtraction are so intimately related that they should be taught at first together. In like manner should be taught the closely related operations of Multiplication and Division.

All written work should be as neat and well arranged as the pupils are capable of doing it. Ill-formed figures and careless arrangement are fruitful sources of error in results.

Pupils while in school should be kept constantly employed. They must *do* if they would learn. Idleness should not be tolerated for a moment. The forward

movement should be frequently halted, and past lessons repeated. It is by repetition that acquisition becomes fixed and sure: the pupil's ignorance never ceases to be the teacher's opportunity.

Pupils should be drilled on all exercises with great care and thoroughness. The primary lessons of the book are presented as suggestive rather than as exhaustive, and the teacher will often find it necessary to supplement them. In such cases the suggestions at the foot of the pages will be helpful.

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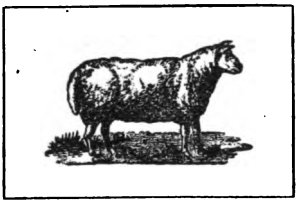
ELEMENTARY ARITHMETIC

PART I.

IDEA OF NUMBER AND PRIMARY PROCESSES.

LESSON I.



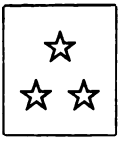
1. How many sheep on the left?
2. How many on the right?
3. How many sheep are one sheep and one sheep?
4. Hold up one finger. Hold up another finger. How many fingers are you now holding up?
5. How many are one and one?
6. One sheep and two sheep are how many sheep? How many are two and one?
7. Instead of writing the words *one*, *two*, and *three*, we may use figures. Now, make them neatly.

1	2	3
One.	Two.	Three.

NOTE.—Drill upon 1, 2, and 3 as *numbers*, not as *figures*. Teach Addition and Subtraction at the same time: 2 and 1 are 3; therefore, 1 from 3 leaves 2, and 2 from 3 leaves 1.

LESSON II.

Four and Combinations.



1. How many hands have you? How many feet?

2. Hold up as many fingers as there are stars on the left.

3. How many stars on the left? How many on the right? Three stars and one star are how many stars?

4. Take four objects from the table. Return one to the table. How many have you now? One taken from four leaves how many?

5. Take four objects. Give two to your teacher. How many have you now? Two taken from four leaves how many?

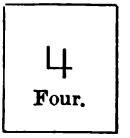
6. Make four short lines on the blackboard. Place your hand over three of the lines. How many do you see? Three from four leaves how many?

7. One star, and one star, and one star, are how many stars? How many ones in three? How many are three ones?

8. How many boys and girls in the class can take objects and show that

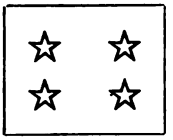
1 and 1 are 2.	1 taken from 2 leaves 1.
1 and 2 are 3.	1 taken from 3 leaves 2.
1 and 3 are 4.	1 taken from 4 leaves 3.
2 and 2 are 4.	2 taken from 3 leaves 1.

To the Teacher.—Exercises similar to the foregoing should be given until pupils can read the combinations—that is, state their value—as promptly as they say "at" when they see the letters a-t.



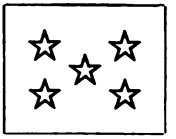
LESSON III.

Five and Combinations.



1. Look at the stars on the left and tell how many?

2. How many twos in four?



3. Look at the stars on the right and tell how many more stars are there than on the left.

4. How many are four and one? Three and two?

5. How many circles are ○ ○ ○ and ○? How many are ○ ○ and ○ ○?

6. Here are some groups of stars. At sight name the number in each.

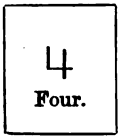
☆ 1	☆☆ 2	☆☆☆ 3	☆☆☆ 3	☆☆☆☆ 4	☆ 1
☆☆ 2	☆☆ 2	☆ 1	☆☆ 2	☆ 1	☆ 1
3	4	4	5	5	2

7. Examine the groups and tell how each is made up. Name the number in each row. Name the number in each group.

8. If you were to use figures for the stars, you would have:

1	2	3	3	4	1
2	2	1	2	1	1

9. We have learned a new number to-day, and must know how to say five with a figure. Here is the figure 5. Try to make it on your slate.



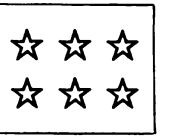
To the Teacher.—Arrange groups of objects from one to five and practice by having pupils name the number in each group. Pupils should recognize at sight the number of objects in each group.

LESSON IV.

Six and Combinations.

1. How many are three and two?

2. How many are five and one?



3. Take three objects from the table. Take three in your other hand. How many have you in both hands? How many are three and three? How many are two and three?

4. Pass one of those in your right hand to your left hand. How many have you in each hand now? How many in both hands? Four and two are how many? Pass one more from your right hand to your left. How many in each hand now? How many in both hands? How many are five and one?

5. Name the numbers in each group.

☆ 1	☆☆ 2	☆☆☆ 3
☆☆☆☆☆ 5	☆☆☆☆ 4	☆☆☆ 3
6	6	6

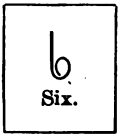
6. Take 5 objects and show that 5 is made up of 4 and 1; 2 and 3.

7. Take 6 objects and show that 6 is made of 5 and 1; 4 and 2; 3 and 3.

8. Take 4 objects and show how many ones are in 4. How many twos.

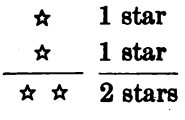
To the Teacher.—These exercises may be made very interesting, but they must be varied in order to hold attention.

Ample preparation should be made for each lesson.

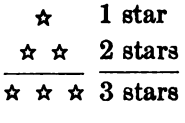


LESSON V.

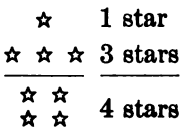
Review.



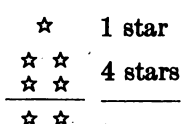
1. How many stars are one star and one star? How many are one and one? How many ones in two? If you take one from two, how many will you have left?



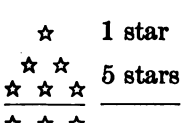
2. One star and two stars are how many? How many are two and one? How many ones in three? If you take one from three, how many will you have left? Two taken from three will leave how many?



3. If you add one star to three stars how many will you have? One and three are how many? One from four leaves how many? Two from four leaves how many? Three from four? How many ones in four? How many twos?



4. Four stars and one star are how many stars? How many are 4 and 1? How many are 2 and 3? How many will be left if you take 1 from 5? 2 from 5? 3 from 5? 4 from 5? How many must be added to 3 to make 5? How many must be added to 4 to make 5?



5. How many are 5 stars and 1 star? How many are 5 and 1? How many are 4 and 2? 3 and 3? Taking 1 from 6, how many are left? 2 from 6? 3 from 6? How many ones in 6? How many twos?

LESSON VI.

Seven and Combinations.

1. How many stars in the upper part of the square? How many in the lower part?

2. How many in the whole square? How many are 6 and 1?

3. Take six objects in your right hand and one in your left. How many are in both hands?

4. Pass one object from right hand to left hand. How many in each hand now? One and one are how many? One from six leaves how many? Name the number of objects in each hand now. How many in both hands? 5 and 2 are how many?

5. Again pass one object from the right hand to the left. How many in each hand now? How many in both hands? How many are 4 and 3?

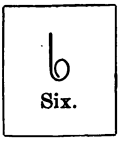
6. Make seven marks on the board. Erase three; how many are left? If 3 be taken from 7, how many will be left?

7. Make seven X's on the board. Erase two of them, and tell how many remain. 7 less 2 are how many?

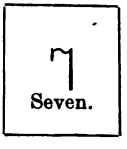
8. Take objects and show how many threes are in six. Show how many twos in six.

9. If you take two threes from seven, how many will be left? How many are two threes? Three twos?

10. Complete the following:



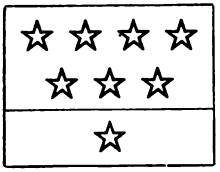
6 and 1 are —. 1 from 7 leaves —.
5 and 2 are —. 2 from 7 leaves —.
4 and 3 are —. 3 from 7 leaves —.



To THE TEACHER. — Pupils should handle the objects.

LESSON VII.

Eight and Combinations.



1. Take from the table one object for every star you see in the upper part of the space. How many have you?

2. Take one more object. How many have you now? 7 and 1 are how many?

3. Place 8 ○'s on the board. Draw a line so as to leave 2 ○'s on one side. How many on the other side of the line. 2 from 8 leaves how many?

4. How many are 6 and 2? 7 and 1?

5. Make a square on the board. Draw a line across the square and place 5 X's in one space and 3 X's in the other. How many X's in the square? 5 and 3 are how many?

6. Take 4 objects in each hand. How many have you in both hands? 4 and 4 are how many? How many fours make eight?

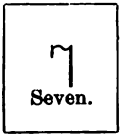
7. With objects show how many 2's in 8.

8. If you had 8 cents, how many apples could you buy at 2 cents apiece?

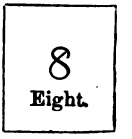
9. If Harry had 8 marbles and gave his little brother 3, how many had he left?

10. How many are two 2's? Three 2's? Four 2's?

11. Complete the following:

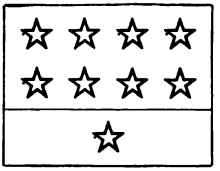


7 and 1 are —. 1 from 8 leaves —.
6 and 2 are —. 2 from 8 leaves —.
5 and 3 are —. 3 from 8 leaves —.
4 and 4 are —. 4 from 8 leaves —.



LESSON VIII.

Nine and Combinations.



1. How many are one and eight?

2. Take two objects in one hand and seven in the other. How many in both hands? 7 and 2 are how many?

3. Take 9 objects. Give your teacher 3. How many have you? 3 from 9 leaves how many? 6 and 3 are how many?

4. Take 9 objects. Give your teacher 4. How many have you? How many are 5 and 4? Taking 4 from 9, how many are left?

5. Take 9 objects. Give your teacher 3 and give one of your classmates 3. How many have you? How many threes in nine?

6. John had 5 rabbits and bought 4 more. How many did he then have?

7. Maud cut 9 roses and gave 3 to Irene. How many did she keep?

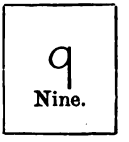
8. Take objects and show that 4 and 4 and 1 are 9.

9. Complete the following:

1 and 8 are —.	1 from 9 leaves —.
2 and 7 are —.	2 from 9 leaves —.
3 and 6 are —.	3 from 9 leaves —.
4 and 5 are —.	4 from 9 leaves —.

3 and 3 and 3 are —.

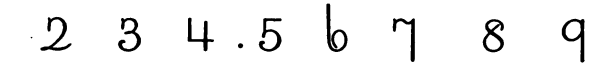
To the Teacher.—Place on the blackboard stars or other simple figures, arranged as at the top of this page, and by drawing vertical lines lead pupils to see that $4 + 4 + 1 = 9$; $2 + 2 + 2 + 2 + 1 = 9$. Use objects freely and vary the questions.



LESSON IX.

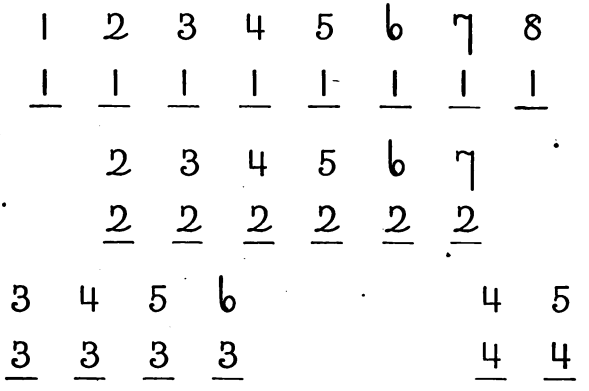
We have learned nine numbers. Here are the figures which we may use when we want to say

one, two, three, four, five, six, seven, eight, nine.



Copy these figures and try to make them correctly.

Here are all the combinations you have learned. Now try to name the sum of each one.



The pupils will name all the combinations that make 9. All that make 8, 7, 6, 5, 4.

To THE TEACHER.—Drill pupils on the table until they can name the sum of each combination at sight. Then have them name the differences. And, lastly, require them to name the sum of each combination and subtract each number in the combination from that sum. Teach numbers, not figures. Use objects in your explanations. This table should be placed upon the board and used for daily drills.

LESSON X.

Sign of Addition.

1. When we want to show that numbers are to be added we may use a cross like this +.

2. It is called "**Plus**" or the **Sign of Addition.**

3. When we find it between numbers it is read "*plus*" or "*and.*" 4 + 2, is read "four plus two" or "four and two."

4. Read these and give the sum of each:

1. 3 + 2.	*7.* 4 + 3.	*13.* 5 + 2.
2. 5 + 3.	*8.* 7 + 1.	*14.* 3 + 3.
3. 4 + 2.	*9.* 5 + 4.	*15.* 5 + 1.
4. 6 + 1.	*10.* 8 + 1.	*16.* 4 + 4.
5. 2 + 2.	*11.* 6 + 2.	*17.* 2 + 1.
6. 7 + 2.	*12.* 3 + 6.	*18.* 1 + 3.

TO THE TEACHER.—Do not permit pupils to count. If the sums cannot be stated promptly, use objects and endeavor to get pupils to think of the number of objects represented by any figure.

LESSON XI.

Problems in Addition.

1. Willie had 5 cents and earned 3 cents. How many cents had he then?

Answer.—He had 5 cents + 3 cents, or 8 cents.

2. Mabel earned 5 cents on Monday and 2 cents on Tuesday. How many cents did she earn in the two days?

3. James paid 4 cents for paper and 5 cents for a pencil. How many cents did he pay for both?

4. Charles had 6 pears and William gave him 2 more. How many had he then?

5. Harry bought a top for 4 cents and a book for 4 cents. How much did he spend?

6. If a sheep costs 6 dollars and a pig costs 3 dollars, how much do both cost?

7. Emma cut 5 roses from one bush and 2 from another. How many did she cut from both bushes?

8. Mary has 3 daisies and Jane has 2. How many have both?

9. How many tops have James and John if each has four? How many 4's in eight?

10. Maud has 2 needles and May has 4. How many have both?

11. Arthur has 4 horses and his cousin has 3. How many horses have both?

12. One ball team made 5 runs and the other made 3 runs. How many did both make?

13. John gathered 2 quarts of berries and Harry gathered 3. How many did both gather?

14. How many are:

1. $4 + 2 + 1$?	*9.* $3 + 3 + 2$?
2. $5 + 2 + 2$?	*10.* $4 + 4 + 1$?
3. $2 + 1 + 5$?	*11.* $5 + 1 + 3$?
4. $3 + 1 + 2$?	*12.* $4 + 2 + 3$?
5. $6 + 1 + 2$?	*13.* $3 + 2 + 4$?
6. $1 + 1 + 4$?	*14.* $4 + 1 + 3$?
7. $2 + 2 + 4$?	*15.* $6 + 1 + 1$?
8. $7 + 1 + 1$?	*16.* $2 + 2 + 1$?

LESSON XII.

Sign of Subtraction.

1. If we want to show that one number is to be taken from another we may use a short line like this —.

2. It is called "**Minus**" or the **Sign of Subtraction.**

3. When we find this sign between numbers, it means that the number on the right is to be taken from the number on the left. 8 — 3, means that 3 must be taken from 8.

4. This sign is read "*minus*" or "*less.*" 7 — 4, is read "Seven minus four," or "Seven less four."

5. Read the following and name the remainders:

1. 4 — 2.	*6.* 6 — 3.	*11.* 9 — 5.
2. 7 — 5.	*7.* 8 — 4.	*12.* 5 — 2.
3. 8 — 3.	*8.* 4 — 3.	*13.* 7 — 3.
4. 6 — 4.	*9.* 6 — 2.	*14.* 9 — 6.
5. 7 — 2.	*10.* 7 — 4.	*15.* 8 — 6.

6. Read, and name results:

1. 4 + 4 — 2.	*6.* 8 + 1 — 4 + 2.
2. 6 — 4 + 5.	*7.* 7 — 2 + 4 — 5.
3. 8 — 6 + 3.	*8.* 6 + 2 + 1 — 7.
4. 7 + 2 — 6.	*9.* 5 + 2 + 2 — 3.
5. 5 + 4 — 6.	*10.* 7 — 4 + 3 + 3.

7. Take 9 objects and show that there are four 2's and a 1 in 9.

8. Show that there are three 2's and a 1 in 7.

9. Show that there are three 3's in 9.

To the Teacher.—Drill pupils on these and similar exercises until they can add and subtract rapidly. At first permit pupils to see the combinations, and afterwards read them to the class and ask for results.

LESSON XIII.

Problems in Subtraction.

1. Oscar had 9 marbles and gave 4 to his brother. How many had he left?

Answer.—He had 9 marbles — 4 marbles, or 5 marbles.

2. Harvey started to walk 8 miles. After he had walked 5 miles, how far had he to go?

3. Fannie had 7 roses and gave her sister 3. How many did Fannie then have?

4. Jacob is 9 years old and his cousin is 6 years old. How much older is Jacob than his cousin?

5. Laura was given 8 words to spell. She missed 2. How many did she spell correctly?

6. James had 9 cents and spent 3 cents for a top. How many cents did he then have?

7. If you had 6 dollars and spent 3 dollars for a hat, how much money would you have left?

8. George earned 5 dollars and spent 3 dollars. How many dollars had he left?

9. There are 7 pears in a basket. How many will be left, if 4 are taken out?

10. Mabel had 5 cents and her brother gave her 4 more. If she then spent 4 cents, how many did she have?

11. How many are:

1. $4 + 5 - 3 + 2 - 3$?
2. $6 - 2 + 5 - 7 - 3$?
3. $5 + 3 - 2 + 3 - 4$?
4. $3 + 3 + 3 - 5 + 2$?
5. $7 + 2 - 1 - 4 + 3$?
6. $6 - 3 + 2 + 4 - 3$?
7. $4 + 4 - 2 + 1 - 5$?
8. $3 + 2 + 2 + 2 - 1$?
9. $1 + 8 - 3 + 1 + 2$?
10. $3 + 2 - 1 + 3 - 7$?

LESSON XIV.

The Number Ten.

1. How many numbers have you learned?

2. Write all these numbers and tell how many figures you use in writing each number.

3. What is the largest number that can be written with one figure?

4. There is another figure to learn. It is called **Naught,** and is made much like one of the letters of the alphabet. Here is the new figure 0. Do not forget its name.

5. Naught does not stand for any number, but it is of use in writing numbers larger than 9.

6. Nine and one are ten. To write ten we must use naught, thus:

10
Ten.

7. How many signs have you learned? Make them and tell what they mean.

8. Before going farther we must learn another sign. It is called the **Sign of Equality,** and is made thus =. It is read "Equals," and it means that what stands on one side of it has the same value as that which stands on the other side. For example, 4 + 6 = 10, is read "Four plus six equals ten," and means that the sum of 4 and 6 is 10.

9. Read and complete:

1. 3 + 7 =	*4.* 7 + 2 =	*7.* 8 − 3 =	*10.* 9 − 3 =
2. 2 + 8 =	*5.* 8 − 5 =	*8.* 10 − 4 =	*11.* 5 + 4 =
3. 9 − 6 =	*6.* 6 + 2 =	*9.* 7 + 3 =	*12.* 8 − 2 =

To the Teacher.—These exercises should be extended. Before passing to the next lesson, review the work gone over, and see that pupils understand how to add and subtract rapidly all numbers up to and including ten.

LESSON XV.

Rapid Work on Combinations.

1. Read and complete:

1. 4 — 2 =	*7.* 3 + 2 =	*13.* 5 — 3 =	*19.* 6 — 4 =
2. 5 + 2 =	*8.* 7 — 5 =	*14.* 7 — 2 =	*20.* 6 + 2 =
3. 1 + 2 =	*9.* 2 + 2 =	*15.* 3 — 1 =	*21.* 4 — 2 =
4. 8 — 3 =	*10.* 5 + 3 =	*16.* 8 — 5 =	*22.* 6 + 3 =
5. 6 — 3 =	*11.* 7 + 3 =	*17.* 10 — 5 =	*23.* 10 — 6 =
6. 10 — 3 =	*12.* 9 — 6 =	*18.* 9 — 4 =	*24.* 10 — 5 =

2. Read and complete:

1. 4 + 3 + 2 — 5 =	*7.* 1 + 5 — 2 + 6 =
2. 2 + 3 + 4 — 1 =	*8.* 5 + 2 — 6 + 4 =
3. 4 — 2 + 3 + 5 =	*9.* 3 + 3 + 3 — 6 =
4. 8 — 5 + 4 + 2 =	*10.* 7 + 3 — 5 + 4 =
5. 6 — 2 + 5 + 2 =	*11.* 6 + 4 — 3 + 1 =
6. 5 + 2 + 2 — 7 =	*12.* 2 + 2 + 3 — 1 =

3. Find the sums of these columns:

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
2	4	6	7	8	3	4
3	3	1	2	1	3	4
2	2	2	1	1	3	1
1	1	1	0	0	1	1

4. What is the value of the following:

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
7	10	9	8	7	6	10	9
— 2	— 6	— 5	— 6	— 3	— 3	— 5	— 7

5. How many are 10 — 7? 10 — 8? 10 — 3? 10 — 2? 10 — 4? 10 — 1? 10 — 9?

LESSON XVI.

Ten to Twenty.

One ten and no ones are ten; written 10

One ten and one are eleven; written 11

One ten and two are twelve; written 12

One ten and three are thirteen; written 13

One ten and four are fourteen; written 14

One ten and five are fifteen; written 15

One ten and six are sixteen; written 16

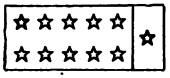
One ten and seven are seventeen; written 17

One ten and eight are eighteen; written 18

One ten and nine are nineteen; written 19

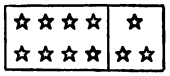
LESSON XVII.

The Number Eleven.



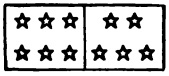
10 and 1.

9 and 2.



8 and 3.

7 and 4.



6 and 5.

1. Take objects and show how many are 6 and 5; 7 and 4; 8 and 3; 9 and 2.

2. Tell how to write eleven.

3. What does the one on the right stand for?

4. What does the one on the left stand for?

5. If Maud had 11 cents and spent 3 cents for apples, how many cents had she left?

6. Rachel had 11 roses in a bouquet and gave 5 to a sick girl. How many did Rachel then have?

7. Oscar earned 11 dollars. If he gave his mother 4 dollars and spent the remainder for a suit, how much did the suit cost?

8. A man having 11 pigs sold 6. How many did he keep?

9. Complete the following:

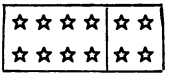
1. $3 + 8 =$	*6.* $11 - 5 =$	*11.* $11 - 1 =$
2. $7 + 4 =$	*7.* $11 - 4 =$	*12.* $11 - 10 =$
3. $11 - 3 =$	*8.* $11 - 9 =$	*13.* $5 + 6 =$
4. $11 - 6 =$	*9.* $10 + 1 =$	*14.* $11 - 7 =$
5. $9 + 2 =$	*10.* $11 - 8 =$	*15.* $11 - 2 =$

LESSON XVIII.

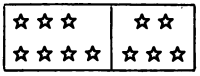
The Number Twelve.

10 and 2.

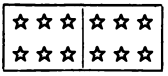
9 and 3.



8 and 4.



7 and 5.



6 and 6.

1. Take twelve objects and show that the number 12 is made up of 6 and 6, of 7 and 5, of 3 and 9, of 4 and 8, of 10 and 2.

2. Write twelve and tell what each figure stands for.

3. Bertha wants to buy a book for twelve cents. If she has a dime, how much does she lack?

4. Ned earned 8 cents on Monday and 4 cents on Tuesday. How much did he earn in the two days?

5. If Ned spent 5 cents on Wednesday, how much had he then?

6. Edna has 5 pins in one cushion and 7 in another. How many pins in both cushions?

7. Elmer put 8 sheep in one field and 4 in another field. How many in both fields?

8. Ruth is 6 years old and Albert is 12 years old. How much older is Albert than Ruth?

9. Complete the following:

1. $12 - 2 =$	*5.* $12 - 6 =$	*9.* $12 - 4 =$
2. $12 - 3 =$	*6.* $12 - 10 =$	*10.* $12 - 8 =$
3. $12 - 9 =$	*7.* $10 + 2 =$	*11.* $9 + 3 =$
4. $7 + 5 =$	*8.* $6 + 6 =$	*12.* $11 + 1 =$

LESSON XIX.

Numbers 13, 14, and 15.

1. Show that thirteen is made up of 10 and 3, 9 and 4, 8 and 5, 7 and 6.

2. How many are:

1. 10 + 3?	*5.* 8 + 5?	*9.* 11 + 2?	*13.* 12 + 1?
2. 6 + 7?	*6.* 9 + 4?	*10.* 13 − 2?	*14.* 13 − 1?
3. 13 − 7?	*7.* 13 − 9?	*11.* 13 − 6?	*15.* 13 − 5?
4. 13 − 4?	*8.* 13 − 3?	*12.* 13 − 2?	*16.* 13 − 10?

3. What does each figure in thirteen stand for?

4. Using objects, show that fourteen is made up of 10 and 4, 8 and 6, 12 and 2, 9 and 5, 7 and 7, 13 and 1.

5. How many are:

1. 10 + 4?	*5.* 14 − 4?	*9.* 14 − 10?	*13.* 11 + 3?
2. 9 + 5?	*6.* 14 − 5?	*10.* 14 − 9?	*14.* 12 + 2?
3. 8 + 6?	*7.* 14 − 6?	*11.* 14 − 8?	*15.* 13 + 1?
4. 7 + 7?	*8.* 14 − 7?	*12.* 14 − 3?	*16.* 14 − 2?

6. What is the value of each figure in fourteen?

7. With objects show that fifteen is composed of 10 and 5, 9 and 6, 8 and 7, 12 and 3, 11 and 4, 13 and 2, 14 and 1.

8. Find the value of:

1. 10 + 5 =	*5.* 15 − 6 =	*9.* 15 − 8 =
2. 9 + 6 =	*6.* 15 − 7 =	*10.* 12 + 3 =
3. 8 + 7 =	*7.* 15 − 10 =	*11.* 11 + 4 =
4. 15 − 5 =	*8.* 15 − 9 =	*12.* 13 + 1 =

9. Do what the signs indicate:

11	15	12	14	15	10	9	8
+ 3	− 4	+ 3	− 7	− 6	+ 5	+ 6	+ 7

LESSON XX.

Numbers 16 to 20.

1. How many tens in 16? In 17? In 18? In 19?

2. What is the units figure in 16? In 17? In 18? In 19?

3. Take 16 objects and show that 8 and 8 are 16. That 9 and 7 are 16. That 10 and 6 are 16. That 11 and 5 are 16, etc.

4. In like manner show the combinations of two figures that make 17.

5. Show the combinations that make 18.

6. Show the combinations that make 19.

7. If you were to take the figure 6 out of 16, what figure should you put in its place? Why?

8. If you take the figure 1 out of the numbers 16, 17, 18, and 19, what remainders would you have?

9. How much less do you make 16, 17, 18, and 19, when you take the 1 away. Why?

10. Rapid work:

1. 16 − 8 =	*10.* 16 + 2 =	*19.* 18 − 7 =
2. 16 − 9 =	*11.* 17 − 10 =	*20.* 18 − 6 =
3. 16 + 3 =	*12.* 17 − 7 =	*21.* 18 − 5 =
4. 16 − 10 =	*13.* 17 − 6 =	*22.* 19 − 8 =
5. 16 − 7 =	*14.* 17 − 5 =	*23.* 19 − 9 =
6. 16 − 6 =	*15.* 18 − 8 =	*24.* 16 + 0 =
7. 16 − 5 =	*16.* 18 − 9 =	*25.* 19 − 10 =
8. 17 − 8 =	*17.* 16 + 1 =	*26.* 19 − 7 =
9. 17 − 9 =	*18.* 18 − 10 =	*27.* 19 − 6 =

NOTE.—This exercise should be extended by the teacher. "Hasten slowly," should be the motto.

LESSON XXI.

Table of Simple Combinations.

1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1
2	3	4	5	6	7	8	9	
2	2	2	2	2	2	2	2	
3	4	5	6	7	8	9		
3	3	3	3	3	3	3		
4	5	6	7	8	9			
4	4	4	4	4	4			
5	6	7	8	9				
5	5	5	5	5				
6	7	8	9					
6	6	6	6					
7	8	9						
7	7	7						
8	9							
8	8							
9								
9								

To THE TEACHER.—The foregoing table shows all the combinations that can be made with the nine digits. Pupils should be drilled upon this table until it is thoroughly learned.

LESSON XXII.

From Lesson XXI. we derive the following:

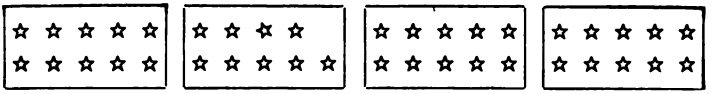
Subtraction Table.

2	3	3	4	5	5	5	5	6	
1	1	2	2	1	2	3	4	1	
6	6	6	6	7	7	7	7	7	7
2	3	4	5	1	2	3	4	5	6
8	8	8	8	8	8	8	9	9	9
1	2	3	4	5	6	7	1	2	3
9	9	9	9	9	10	10	10	10	10
4	5	6	7	8	1	2	3	4	5
10	10	10	10	11	11	11	11	11	11
6	7	8	9	2	3	4	5	6	7
11	11	12	12	12	12	12	12	12	13
8	9	3	4	5	6	7	8	9	4
13	13	13	13	13	14	14	14	14	14
5	6	7	8	9	5	6	7	8	9
15	15	15	15	16	16	16	17	17	18
6	7	8	9	7	8	9	8	9	9

To THE TEACHER.—This table should be learned thoroughly. Lead pupils to see that, if 7 and 6 are 13, $13 - 6 = 7$, and $13 - 7 = 6$.

LESSON XXIII.

Twenty to Thirty.



10 + 9 = 19. 10 + 10 = 20.

1. In 19 what does 1 stand for? What does 9 stand for?

2. If you increase 19 by 1, do you add 1 to the 1 or to the 9?

3. Adding 1 to 9 will make how many? Adding 1 to 19, then, will make how many 10's?

4. In 20 what figure shows that there are two 10's in twenty? Why is naught used?

5. Count twenty objects and show that there are two 10's in 20. Four 5's. Five 4's.

6. How many are 2 times 10? 4 times 5? 5 times 4?

7. 2 tens and 1 = **twenty-one**, written 21

2 tens and 2 = **twenty-two**, written 22

2 tens and 3 = **twenty-three**, written 23

2 tens and 4 = **twenty-four**, written 24

2 tens and 5 = **twenty-five**, written 25

2 tens and 6 = **twenty-six**, written 26

2 tens and 7 = **twenty-seven**, written 27

2 tens and 8 = **twenty-eight**, written 28

2 tens and 9 = **twenty-nine**, written 29

LESSON XXIV.

Thirty to One Hundred.

1. Three tens are **thirty**, written 30
2. Write the numbers from thirty to thirty-nine.
3. Four tens are **forty**, written 40
4. Write the numbers from forty to forty-nine.
5. Five tens are **fifty**, written 50
6. Write the numbers from fifty to fifty-nine.
7. Six tens are **sixty**, written 60
8. Write the numbers from sixty to sixty-nine.
9. Seven tens are **seventy**, written 70
10. Write the numbers from seventy to seventy-nine.
11. Eight tens are **eighty**, written 80
12. Write the numbers from eighty to eighty-nine.
13. Nine tens are **ninety**, written 90
14. Write the numbers from ninety to ninety-nine.
15. If one be added to ninety-nine, we shall have ten 10's or **one hundred**, written 100

16. How many tens in 50? In 30? In 40? In 60? In 80? In 90? In 70? In 100?

17. How many are:

1. 70 + 5?	*8.* 22 + 2?	*15.* 92 + 4?
2. 60 + 8?	*9.* 36 + 2?	*16.* 75 + 5?
3. 40 + 7?	*10.* 41 + 8?	*17.* 28 + 2?
4. 90 + 7?	*11.* 62 + 7?	*18.* 65 + 5?
5. 50 + 3?	*12.* 56 + 4?	*19.* 83 + 4?
6. 30 + 6?	*13.* 72 + 6?	*20.* 36 + 5?
7. 20 + 9?	*14.* 84 + 4?	*21.* 90 + 9?

NOTE.—Numbers from 100 to 1000 should be taught in succeeding lessons.

LESSON XXV.

Multiplication.

1. If you should want to know how many cents 5 apples would cost at 2 cents apiece, you might say $2 + 2 + 2 + 2 + 2 = 10$. But there is a shorter way. If we add 2 and 2 and find the sum to be 4, or if we add 2 and 2 and 2 and find the sum to be 6, and so on, we can write down these results in the form of a table and then commit the table to memory. In the table the sign $\times$ is read "times."

$2 \times 1 = 2.$	$2 \times 4 = 8.$	$2 \times 7 = 14.$
$2 \times 2 = 4.$	$2 \times 5 = 10.$	$2 \times 8 = 16.$
$2 \times 3 = 6.$	$2 \times 6 = 12.$	$2 \times 9 = 18.$

2. By using objects prove that each result in the table is correct.

3. If Harry bought 2 books at 9 cents apiece, how much did he spend?

4. What will 2 oranges cost if one orange is worth 5 cents?

5. At 8 cents a quart, what will 2 quarts of berries cost?

6. Edna bought 2 yards of crash at 8 cents a yard. How much did she spend?

7. William walked 7 miles a day for 2 days. How far did he walk?

8. Rapid work:

1. $2 \times 1 + 2 =$	*5.* $4 + 8 - 6 =$	*9.* $2 \times 8 + 4 =$
2. $2 \times 4 - 2 =$	*6.* $5 + 7 + 3 =$	*10.* $2 \times 4 + 8 =$
3. $2 \times 5 + 5 =$	*7.* $7 + 8 - 5 =$	*11.* $2 \times 7 - 4 =$
4. $2 \times 6 - 2 =$	*8.* $9 + 7 - 3 =$	*12.* $2 \times 9 + 6 =$

LESSON XXVI.

Multiplication by Three.

$3 \times 1 = 3.$	$3 \times 4 = 12.$	$3 \times 7 = 21.$
$3 \times 2 = 6.$	$3 \times 5 = 15.$	$3 \times 8 = 24.$
$3 \times 3 = 9.$	$3 \times 6 = 18.$	$3 \times 9 = 27.$

1. How many miles will a horse trot in 3 hours if he can trot 5 miles in one hour?

2. At 6 cents a yard, what will 3 yards of ribbon cost?

3. What will 9 pencils cost at 3 cents apiece?

4. Jane is 7 years old and her sister is 3 times as old. How old is her sister?

5. If one orange costs 4 cents, what is the cost of 3 oranges?

6. Edith has 3 rose-bushes. If she cuts 8 roses from each bush, how many roses will she have?

7. If a boy walks 3 miles an hour, how far can he walk in 9 hours?

8. Rapid work:

1.	$8 + 3.$	$18 + 3.$	$28 + 3.$	$38 + 3.$
2.	$9 + 5.$	$19 + 5.$	$29 + 5.$	$39 + 5.$
3.	$8 + 7.$	$18 + 7.$	$28 + 7.$	$38 + 7.$
4.	$6 + 5.$	$16 + 5.$	$26 + 5.$	$36 + 5.$
5.	$5 + 8.$	$15 + 8.$	$25 + 8.$	$35 + 8.$
6.	$13 - 8.$	$23 - 8.$	$33 - 8.$	$43 - 8.$
7.	$15 - 9.$	$25 - 9.$	$35 - 9.$	$45 - 9.$
8.	$14 - 5.$	$24 - 5.$	$34 - 5.$	$44 - 5.$
9.	$11 - 7.$	$21 - 7.$	$31 - 7.$	$41 - 7.$
10.	$16 - 8.$	$26 - 8.$	$36 - 8.$	$46 - 8.$
11.	$3 \times 9.$	$3 \times 8.$	$3 \times 6.$	$3 \times 7.$

LESSON XXVII.

Division by Two and Three.

1. 2 + 2 are how many? 2 times 2 are how many? How many 2's in 4?

2. If we take 6 objects and arrange them in three parts, 2 in each, we are dividing.

3. Divide 8 into 2 equal parts. This is dividing by 2. Divide 8 by 4. Did you find how many 4's in 8?

4. How many 3's in 9? With objects show that your answer is correct?

5. Since 3 times 4 = 12, how many 4's in 12? How many 3's?

6. Turn to Lesson XXV., examine the Multiplication Table, and tell how many 2's in 4.

7. How many 2's in 12? In 16? In 14? In 18? In 10? In 8? In 6?

8. To indicate Division we use this sign ÷. It is read "divided by."

9. Complete the following:

1. 18 ÷ 2 =	*4.* 4 ÷ 2 =	*7.* 14 ÷ 2 =
2. 12 ÷ 2 =	*5.* 6 ÷ 2 =	*8.* 16 ÷ 2 =
3. 10 ÷ 2 =	*6.* 2 ÷ 2 =	*9.* 8 ÷ 2 =

10. Examine the table in Lesson XXVI., and complete the following:

1. 6 ÷ 3 =	*4.* 15 ÷ 3 =	*7.* 18 ÷ 3 =
2. 9 ÷ 3 =	*5.* 12 ÷ 3 =	*8.* 27 ÷ 3 =
3. 3 ÷ 3 =	*6.* 21 ÷ 3 =	*9.* 24 ÷ 3 =

TO THE TEACHER.—Lead the pupils to understand Division by directing attention to the fact that the expression 3 × 8 = 24 shows not only the product of 3 and 8, but it shows how many 8's and how many 3's in 24.

LESSON XXVIII.

Multiplication and Division.

1. With objects prove the results in the following table and then commit the table:

$4 \times 1 = 4.$	$4 \times 4 = 16.$	$4 \times 7 = 28.$
$4 \times 2 = 8.$	$4 \times 5 = 20.$	$4 \times 8 = 32.$
$4 \times 3 = 12.$	$4 \times 6 = 24.$	$4 \times 9 = 36.$

2. How many 4's in 8? In 36? In 24? In 20? In 16? In 28? In 32?

3. State results:

1. $4 \times 5 - 3.$ *4.* $3 \times 7 - 4.$ *7.* $12 \div 3 + 6.$

2. $3 \times 6 + 5.$ *5.* $4 \times 6 + 6.$ *8.* $36 \div 4 + 3.$

3. $4 \times 9 - 6.$ *6.* $9 \div 3 + 6.$ *9.* $28 \div 4 - 2.$

4. What will each boy get if 32 cents be divided equally among 4 boys?

5. What is the cost of 4 two-cent postage stamps?

6. At 9 cents a pound, what will 4 pounds of raisins cost?

7. If a man earns 4 dollars a day, how much does he earn in 4 days?

8. Divide 24 apples among 4 boys. How many apples will each boy get?

9. What is the cost of 4 tops at 5 cents apiece?

10. How many eggs at 4 cents apiece can be bought for 28 cents?

Slate Work.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
$2\overline{)46}$	$3\overline{)36}$	$4\overline{)84}$	$2\overline{)48}$	$3\overline{)96}$	$3\overline{)63}$

LESSON XXIX.

Multiplication.	Division.
$5 \times 1 = 5$;	therefore, $5 \div 5 = 1$
$5 \times 2 = 10$;	therefore, $10 \div 5 = 2$, and $10 \div 2 = 5$
$5 \times 3 = 15$;	therefore, $15 \div 5 = 3$, and $15 \div 3 = 5$
$5 \times 4 = 20$;	therefore, $20 \div 5 = 4$, and $20 \div 4 = 5$
$5 \times 5 = 25$;	therefore, $25 \div 5 = 5$, and $25 \div 5 = 5$
$5 \times 6 = 30$;	therefore, $30 \div 5 = 6$, and $30 \div 6 = 5$
$5 \times 7 = 35$;	therefore, $35 \div 5 = 7$, and $35 \div 7 = 5$
$5 \times 8 = 40$;	therefore, $40 \div 5 = 8$, and $40 \div 8 = 5$
$5 \times 9 = 45$;	therefore, $45 \div 5 = 9$, and $45 \div 9 = 5$

Rapid Work.

1. $4 \times 4 \div 2 + 4 - 2$.
2. $5 \times 4 \div 2 + 5 - 3$.
3. $4 \times 8 \div 4 - 4 + 4$.
4. $5 \times 3 + 4 - 6 + 7$.
5. $6 \times 4 \div 3 + 7 - 3$.
6. $5 \times 7 - 3 + 6 + 3$.
7. $5 \times 9 + 5 + 8 - 6$.
8. $5 \times 5 + 5 - 7 - 8$.
9. $5 \times 3 + 8 + 7 - 10$.
10. $5 \times 8 - 8 + 6 - 10$.

Slate Work.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
81	42	24	53	62	91
$\times 5$	$\times 3$	$\times 2$	$\times 3$	$\times 4$	$\times 5$

(7.)	(8.)	(9.)	(10.)	(11.)	(12.)
$5\overline{)55}$	$3\overline{)69}$	$5\overline{)45}$	$2\overline{)86}$	$4\overline{)80}$	$5\overline{)50}$

(13.)	(14.)	(15.)	(16.)	(17.)
$3\overline{)306}$	$5\overline{)255}$	$4\overline{)324}$	$4\overline{)288}$	$3\overline{)279}$

NOTE.—The teacher should explain with great care.

LESSON XXX.

Multiplication and Division by Six.

$6 \times 1 = 6.$	$6 \times 4 = 24.$	$6 \times 7 = 42.$
$6 \times 2 = 12.$	$6 \times 5 = 30.$	$6 \times 8 = 48.$
$6 \times 3 = 18.$	$6 \times 6 = 36.$	$6 \times 9 = 54.$

From the foregoing table give results to the following, and state reasons for results:

1. $18 \div 6 =$	*5.* $12 \div 2 =$	*9.* $54 \div 6 =$
2. $30 \div 5 =$	*6.* $48 \div 6 =$	*10.* $30 \div 6 =$
3. $36 \div 6 =$	*7.* $24 \div 6 =$	*11.* $54 \div 9 =$
4. $18 \div 3 =$	*8.* $48 \div 8 =$	*12.* $42 \div 6 =$

To THE TEACHER.—Have pupils commit the Multiplication Table, and show them that the product of two numbers divided by one of the numbers will give the other. Be patient; do not forget your own first efforts to master these tables.

Remember that "Repetition is the mother of all learning."

LESSON XXXI.

Multiplication and Division by Seven.

$7 \times 1 = 7.$	$7 \times 4 = 28.$	$7 \times 7 = 49.$
$7 \times 2 = 14.$	$7 \times 5 = 35.$	$7 \times 8 = 56.$
$7 \times 3 = 21.$	$7 \times 6 = 42.$	$7 \times 9 = 63.$

Give results and state reasons, as in the preceding lesson:

1. $42 \div 7 =$	*5.* $28 \div 7 =$	*9.* $49 \div 7 =$
2. $21 \div 7 =$	*6.* $63 \div 7 =$	*10.* $56 \div 7 =$
3. $14 \div 7 =$	*7.* $35 \div 7 =$	*11.* $42 \div 6 =$
4. $28 \div 4 =$	*8.* $63 \div 9 =$	*12.* $14 \div 2 =$

LESSON XXXII.

Multiplication and Division by Eight.

$8 \times 1 = 8.$	$8 \times 4 = 32.$	$8 \times 7 = 56.$
$8 \times 2 = 16.$	$8 \times 5 = 40.$	$8 \times 8 = 64.$
$8 \times 3 = 24.$	$8 \times 6 = 48.$	$8 \times 9 = 72.$

Complete the following as in former lessons:

1. $40 \div 8 =$	*6.* $24 \div 8 =$	*11.* $16 \div 8 =$
2. $72 \div 8 =$	*7.* $16 \div 2 =$	*12.* $64 \div 8 =$
3. $48 \div 6 =$	*8.* $64 \div 8 =$	*13.* $40 \div 5 =$
4. $32 \div 4 =$	*9.* $72 \div 9 =$	*14.* $32 \div 8 =$
5. $56 \div 7 =$	*10.* $48 \div 8 =$	*15.* $56 \div 8 =$

Slate Work.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
$2\overline{)64}$	$2\overline{)48}$	$3\overline{)66}$	$2\overline{)40}$	$4\overline{)48}$	$2\overline{)648}$

LESSON XXXIII.

Multiplication and Division by Nine.

$9 \times 1 = 9.$	$9 \times 4 = 36.$	$9 \times 7 = 63.$
$9 \times 2 = 18.$	$9 \times 5 = 45.$	$9 \times 8 = 72.$
$9 \times 3 = 27.$	$9 \times 6 = 54.$	$9 \times 9 = 81.$

Complete the following:

1. $54 \div 9 =$	*6.* $45 \div 9 =$	*11.* $36 \div 4 =$
2. $81 \div 9 =$	*7.* $27 \div 9 =$	*12.* $18 \div 9 =$
3. $72 \div 8 =$	*8.* $18 \div 2 =$	*13.* $54 \div 6 =$
4. $45 \div 5 =$	*9.* $36 \div 9 =$	*14.* $72 \div 9 =$
5. $63 \div 9 =$	*10.* $64 \div 8 =$	*15.* $63 \div 7 =$

LESSON XXXIV.

Miscellaneous Exercises.—Slate Work.

1. Add:

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
2	3	4	2	3	4	2	1
3	5	5	6	8	5	2	5
4	2	6	4	2	6	3	8
5	1	3	5	1	7	3	6

(9.)	(10.)	(11.)	(12.)	(13.)	(14.)	(15.)	(16.)
9	8	9	3	10	12	14	19
3	6	8	9	3	2	6	1
2	3	3	4	5	1	5	7
1	2	1	6	4	8	5	2

2. Subtract:

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
63	89	34	87	59	99	78
42	73	13	35	27	35	18

3. Multiply:

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
12	14	21	34	22	21	32
2	2	3	2	4	5	4

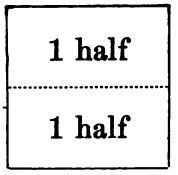
(8.)	(9.)	(10.)	(11.)	(12.)	(13.)	(14.)
15	24	36	29	34	52	64
2	3	4	6	6	7	8

To THE TEACHER.—These simple exercises should be extended until pupils acquire facility in performing the operations. CAUTION: Do not increase difficulties too rapidly; carefully explain every step.

LESSON XXXV.

Common Fractions.

HALVES.



1. If you wanted to share an apple equally with your brother, into how many parts would you cut the apple?

2. What part of the apple would your brother get? What part would you keep?

3. How many halves in one apple? How many halves in 1?

4. How many halves in 2? In 3? In 4? In 5?

5. To find one-half of a number divide by 2.

6. One-half is written $\frac{1}{2}$.

7. Find $\frac{1}{2}$ of 4, 6, 8, 12, 14, 16, 10, 20, 28.

8. Find $\frac{1}{2}$ of 5, 7, 9, 13, 15, 11, 17, 19, 21.

9. Divide 16 oranges between 2 boys.

10. If two slates cost 22 cents, what is the cost of one slate?

11. A boy had 14 cents, and lost one-half of them. How many had he left?

12. If you had 18 cents and should lose one-half of your money, how many oranges at 3 cents apiece could you buy for the remainder?

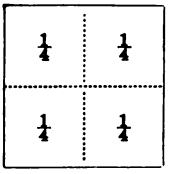
13. John and James caught 12 fish, and divided them equally. How many did each get?

14. If you had 1 melon, to how many persons could you give $\frac{1}{2}$ melon?

To the Teacher.—The teacher will explain No. 8, and by the use of objects both Nos. 7 and 8.

LESSON XXXVI.

FOURTHS.



1. If a square be divided into four equal parts, what is one part called?

2. How many fourths in anything? How many fourths in 1?

3. How do you write one-fourth?

4. What part of one-half is one-fourth?

5. How many fourths in one-half?

6. How many fourths in one dollar? In 2 dollars? In 4 dollars? In 5 dollars?

7. To find one-fourth, into how many parts do you divide a thing?

8. What do you divide by to find $\frac{1}{4}$ of a number?

9. What is $\frac{1}{4}$ of 4? Of 12? Of 20? Of 24?

10. If 4 boys share one dollar equally, what part of the dollar does each boy get?

11. What is one-third of 3 apples? What is 2 thirds of 3 apples? 3 thirds?

12. If 12 roses be equally divided among 3 girls, how many will each have?

13. What is $\frac{1}{4}$ of 28? Of 32? Of 36? Of 16?

14. Take an apple and show how you would find $\frac{1}{2}$ of an apple. How $\frac{1}{4}$ of an apple. Three-fourths.

15. What is $\frac{1}{4}$ of 9? Of 13? Of 17? Of 21?

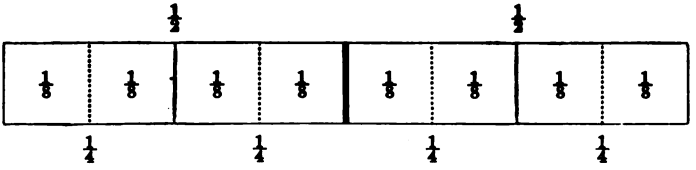
16. Find $\frac{3}{4}$ of 4, 8, 12, 16, 20, 24, 28.

17. Two-fourths of an orange is what part of it?

TO THE TEACHER.—Make many simple problems requiring the finding of one-fourth.

LESSON XXXVII.

EIGHTHS.



1. How many halves are shown above? How many fourths?

2. Into how many parts is each fourth divided?

3. Into how many parts is the whole space divided?

4. If anything be divided into 8 equal parts, what is one part called?

5. How do you write one-eighth? Three-eighths?

6. How many eighths in anything?

7. How many eighths in one-half of anything?

8. How many eighths in one-fourth of anything?

9. How do you find one-eighth of an apple? Take an apple and show how it is done.

10. How do you find $\frac{1}{8}$ of 16 objects? Show how with objects.

11. How would you find $\frac{1}{8}$ of any number?

12. What is $\frac{1}{8}$ of 22? Of 40? Of 80? Of 32?

13. Find $\frac{3}{8}$ of 8, 16, 24, 32, 40, 48, 56.

14. How many eighths in 1? In 2, 3, 4, 5, and 6?

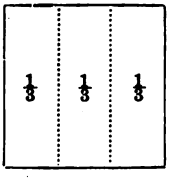
15. What is $\frac{1}{8}$ of 17, 25, 41, 49, 57?

16. What part of a number do you get when you divide by 2? By 4? By 8?

17. Find $\frac{5}{8}$ of 16, 32, 24, 40, 56, 72, 80, and 48.

LESSON XXXVIII.

THIRDS.



1. Into how many equal parts is the square divided?

2. What is each part called?

3. How many thirds in one square?

4. If you should spend $\frac{2}{3}$ of your money, how many thirds would you have?

5. How do you find one-third of an orange?

6. How many thirds in 1? In 2? In 3? In 4?

7. How do you find $\frac{1}{3}$ of a number?

8. What is one-third of 6, 9, 15, 18, 12, 24, 30?

9. How do you find two-thirds of a number?

10. Find $\frac{2}{3}$ of 6, 12, 15, 9, 18, 21, 30, 24, 27.

11. If Edna is 15 years old and Edith is two-thirds as old, how old is Edith?

12. Harry had 21 problems to solve. He solved $\frac{2}{3}$ of the number. How many did he miss?

13. Walter started to ride 36 miles on his bicycle. After he had gone $\frac{2}{3}$ of the distance, how far had he to go?

14. A boy having 33 cents, spent $\frac{2}{3}$ of his money. How much did he spend? How much had he left?

15. What is the difference between one-third and one-fourth of 24?

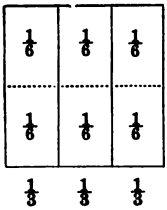
16. At 12 dollars a barrel, what will $\frac{2}{3}$ of a barrel of fish cost?

17. What is $\frac{1}{3}$ of 7, 9, 13, 16, 19, 22, 25?

18. Albert has $\frac{1}{3}$ of 27 dollars and Roscoe has four times as much. How many dollars has Roscoe?

LESSON XXXIX.

SIXTHS.



1. If anything be divided into six equal parts, each part is called one-sixth. Written $\frac{1}{6}$.

2. How many thirds in the square?

3. Into how many parts is each third divided?

4. How many sixths in the square? How many sixths in each third?

5. Does $\frac{1}{3} = \frac{2}{6}$? $\frac{2}{3}$ = how many sixths?

6. Draw a square and show that $\frac{1}{2} = \frac{3}{6}$.

7. How do you find one-sixth of a thing?

8. How do you find one-sixth of a number?

9. State $\frac{1}{6}$ of 6, 18, 24, 12, 30, 42, 48, 36.

10. How many sixths in 1, 3, 2, 4, 6, and 5?

11. State $\frac{5}{6}$ of 6, 24, 12, 18, 30, 36, 42, and 48.

12. Five-sixths of 24 years is twice the age of Charles. How old is Charles?

13. Maud has $\frac{5}{6}$ as many roses as Jane. How many has Maud if Jane has 30 roses?

14. If William has $\frac{5}{6}$ of 18 cents, how much more does he need to buy 10 two-cent stamps?

15. If one man earns 42 dollars and another earns $\frac{4}{6}$ as much, how much will the second man earn?

16. Complete the following:

1. $\frac{1}{6}$ of 60 =	*5.* $\frac{5}{6}$ of 48 =	*9.* $\frac{3}{4}$ of 36 =
2. $\frac{1}{6} \times 5$ =	*6.* $\frac{1}{8} \times 7$ =	*10.* $\frac{5}{6} \times 4$ =
3. $\frac{2}{3}$ of 27 =	*7.* $\frac{1}{2}$ of 84 =	*11.* $\frac{1}{3}$ of 96 =
4. $\frac{3}{4} \times 10$ =	*8.* $\frac{4}{5} \times 12$ =	*12.* $\frac{6}{6}$ of 84 =

LESSON XL.

FIFTHS AND TENTHS.

1. If a pear be divided into 5 equal parts, each part is called one-fifth; written $\frac{1}{5}$.

2. What are two parts called? Three parts? Four parts?

3. Write two-fifths, three-fifths, four-fifths.

4. How is one-fifth of a number found? Two-fifths? Three-fifths? Four-fifths?

5. Find $\frac{1}{5}$ of 10, 20, 30, 40, 15, 25, 50, 35, 45.

6. Complete the following:

1. $\frac{2}{5}$ of 20 =	4. $\frac{2}{5}$ of 35 =	7. $\frac{3}{5}$ of 25 =
2. $\frac{3}{5}$ of 40 =	5. $\frac{4}{5}$ of 45 =	8. $\frac{2}{5}$ of 30 =
3. $\frac{4}{5}$ of 15 =	6. $\frac{3}{5}$ of 50 =	9. $\frac{4}{5}$ of 10 =

7. If anything be divided into 10 equal parts, each part is called one-tenth; written $\frac{1}{10}$.

8. Write three-tenths, five-tenths, seven-tenths, nine-tenths.

9. Can you show that $\frac{2}{10} = \frac{1}{5}$, $\frac{4}{10} = \frac{2}{5}$, $\frac{6}{10} = \frac{3}{5}$?

10. How many tenths in 1, 2, 3, 5, 7, 6, 9, 8?

11. Name $\frac{1}{10}$ of 40, 50, 80, 100, 70, 60, 30.

12. $\frac{7}{10}$ of 60 are how many times 3?

13. A drover, having 100 sheep, sold $\frac{3}{10}$ of them to one man and $\frac{7}{10}$ to another. How many did he have left?

14. Complete the following:

1. $\frac{3}{10}$ of 30 =	4. $\frac{9}{10}$ of 60 =	7. $\frac{7}{10}$ of 100 =
2. $\frac{7}{10}$ of 80 =	5. $\frac{7}{10}$ of 20 =	8. $\frac{3}{10}$ of 100 =
3. $\frac{9}{10}$ of 70 =	6. $\frac{8}{10}$ of 40 =	9. $\frac{9}{10}$ of 100 =

To THE TEACHER.—Simple problems should be given here.

LESSON XLI.

General Review.

1. Make all the signs that you have learned and explain their use.

2. How many processes have you learned?

3. Make a simple problem in Addition and solve it.

4. Make a Subtraction problem, solve and explain it.

5. Repeat the Multiplication Table.

6. Show how Division may be learned from the Multiplication Table.

7. Tell what is meant by one-half. One-third. One-fourth. One-fifth. One-sixth. One-seventh. One-eighth. One-ninth. One-tenth.

8. Show how many halves, thirds, fourths, fifths, sixths, sevenths, eighths, ninths, and tenths in 1.

9. How do you find one-sixth of a number? One-tenth? One-third? One-fifth?

10. Divide each of the following numbers by 7 and by 9 and name the remainders, if any: 63, 81, 49, 54, 90, 42, 38.

11. What parts of the numbers did you find in No. 10?

12. Complete the following:

1. $2 \times 3 + 5 - 1 =$

2. $8 \div 2 - 2 + 9 =$

3. $9 \times 4 \div 3 + 8 =$

4. $6 \times 2 \times 2 + 6 =$

5. $5 \times 4 \times 4 \div 2 =$

6. $18 \div 3 + 6 - 5 =$

7. $\frac{9}{10}$ of $90 - 1 + 20 =$

8. $18 \div 3 + 20 - 13 =$

9. $30 \div 3 - 10 + 9 =$

10. $\frac{1}{3}$ of $36 \times 4 - 9 =$

11. $\frac{3}{4}$ of $24 \div 2 + 8 =$

12. $\frac{5}{6}$ of $36 - 9 + 6 =$

13. $27 + 3 - 16 \div 2 =$

14. $\frac{6}{7}$ of $42 + 6 + 4 \times 2 =$

LESSON XLII.

State results promptly:

1. $19 + 9 =$
2. $\frac{1}{2}$ of $18 =$
3. $27 - 9 =$
4. $42 \div 7 =$
5. $9 \times 8 =$
6. $63 \div 9 =$
7. $49 + 9 =$
8. $\frac{3}{8}$ of $24 =$
9. $\frac{5}{6}$ of $18 =$
10. $9 \times 9 =$
11. $90 \div 10 =$
12. $37 - 8 =$
13. $\frac{1}{4}$ of $36 =$
14. $\frac{3}{4}$ of $36 =$
15. $81 \div 9 =$
16. $76 + 9 =$
17. $21 - 7 =$
18. $69 + 9 =$
19. $63 \div 7 =$
20. $\frac{1}{10}$ of $90 =$
21. $\frac{7}{10}$ of $90 =$
22. $14 + 19 =$
23. $18 - 11 =$
24. $72 \div 8 =$
25. $\frac{1}{3}$ of $27 =$
26. $\frac{2}{3}$ of $36 =$
27. $\frac{3}{3}$ of $18 =$
28. $45 \div 5 =$
29. $\frac{1}{5}$ of $45 =$
30. $27 \times \frac{1}{3} =$
31. $7 \times 7 =$
32. $\frac{1}{7}$ of $49 =$
33. $8 \times 8 =$
34. $3 \times 12 =$
35. $24 - 9 =$
36. $64 \div 8 =$
37. $33 \div 3 =$
38. $\frac{1}{5}$ of $25 =$
39. $\frac{4}{5}$ of $25 =$
40. $6 \times 6 =$
41. $\frac{1}{6}$ of $36 =$
42. $\frac{5}{6}$ of $36 =$
43. $28 + 9 =$
44. $28 \div 7 =$
45. $36 \div 6 =$
46. $49 \div 7 =$
47. $72 \div 9 =$
48. $9 \times 8 =$
49. $21 \div 7 =$
50. $\frac{1}{7}$ of $21 =$
51. $\frac{3}{7}$ of $21 =$
52. $29 + 8 =$
53. $37 - 29 =$
54. $\frac{5}{8}$ of $24 =$
55. $18 \div 9 =$
56. $27 - 9 =$
57. $37 + 7 =$
58. $18 \div 6 =$
59. $\frac{1}{6}$ of $18 =$
60. $\frac{7}{6}$ of $18 =$
61. $99 + 9 =$
62. $99 \div 9 =$
63. $\frac{1}{3}$ of $99 =$
64. $80 \div 10 =$
65. $\frac{1}{10}$ of $80 =$
66. $6 \times 4 =$
67. $4 \times 6 =$
68. $\frac{3}{8}$ of $24 =$
69. $\frac{3}{10}$ of $40 =$
70. $\frac{1}{6} \times 42 =$
71. $42 \div 6 =$
72. $59 + 9 =$
73. $69 + 9 =$
74. $7 \times 8 =$
75. $56 \div 8 =$
76. $56 \div 7 =$
77. $\frac{1}{7}$ of $56 =$
78. $\frac{1}{8}$ of $56 =$
79. $9 \times 4 =$
80. $36 \div 9 =$
81. $\frac{1}{3}$ of $36 =$

This page may be divided into two or more lessons.

ELEMENTARY ARITHMETIC

PART II.

DEFINITIONS.

1. A **Unit** is a single thing; as, *a book, a man, an hour.*

2. A **Number** is a unit, or a collection of units; as, *one hat, ten birds, twenty-five dollars.*

3. An **Abstract Number** is one whose unit is not named; as, 3, 10, 21.

4. A **Concrete Number** is one whose unit is named; as, 6 *pens*, 7 *days*, 10 *dollars.*

All abstract numbers have the same unit; hence, the value of an abstract number depends entirely upon the number of its units. Concrete numbers do not all have units of the same value, and therefore the value of a concrete number depends both upon the value of its unit and upon the times its unit is repeated. A five-dollar bill is more valuable than a two-dollar bill, for five repetitions of the unit, one dollar, give a greater value than two repetitions. Five dozen eggs is a greater number than five eggs, for, while the repetitions of the unit are the same, the unit, one dozen eggs, is greater than the unit, one egg. Hence, the **Unit of a Number** is the *measure* of the number, and determines its value.

5. **Arithmetic** explains numbers and teaches methods of using them.

6. In writing numbers, characters called **Figures** are used, and also certain **Capital Letters.**

7. The writing of numbers with figures or with letters is called **Notation.**

8. The reading of numbers is called **Numeration.**

EXERCISES.

1. What is a unit? How many units are there in eight?

2. Compare eight pounds with eight pounds. Have these numbers the same unit? Has each the same number of units? Then, if you are asked why eight pounds equal eight pounds, what answer can you give?

3. In like manner compare the following: Ten ounces and six ounces. Ten dozen eggs and ten eggs. Twelve feet and ten inches.

4. Name the unit in each of the following, and tell which are concrete and which abstract:

1. Four.
2. Nine ships.
3. Two sailors.
4. Seven.
5. Four guns.
6. Twelve dollars.
7. Eleven.
8. Three eagles.
9. Twenty.
10. Thirty horses.

NOTATION AND NUMERATION.

DEFINITIONS.

1. The system of notation employing *figures* is called the **Arabic System**; that employing *letters* is called the **Roman System.**

The one was introduced into Europe by the Arabs; the other was used by the ancient Romans.

2. The Arabic system employs ten figures to represent numbers.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

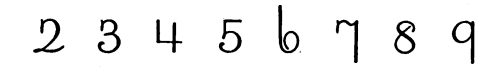
Names: Naught, one, two, three, four, five, six, seven, eight, nine.

Naught is also called **zero** and **cipher**. The rest are called **significant figures**.

Because we have ten fingers, these ten figures are sometimes called **digits**. [Latin, *digitus*, finger.]

3. The following are correct forms for these digits:

Slant Script Figures.



Vertical Script Figures.

EXERCISES.

Make the best Arabic forms you can for the following:

1. Naught, one, two, three, four, five, six, seven, eight.
2. Two, three, four, five, six, seven, eight, nine.
3. Three, four, five, six, seven, eight, nine, naught.
4. Four, five, six, seven, eight, nine, naught, one.
5. Five, six, seven, eight, nine, naught, one, two.
6. Six, seven, eight, nine, naught, one, two, three.
7. Seven, eight, nine, naught, one, two, three, four.
8. Eight, nine, cipher, one, two, three, four, five.
9. Nine, zero, one, two, three, four, five, six.
10. Naught, one, two, three, four, five, six, seven.

Numbers from Ten to Twenty.

1. The number next above nine is called *ten* (10).

2. Notation forms numbers into groups of *ten each.* The first group is: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

3. The second group of ten numbers is formed as follows:

11, named *eleven*, one and ten.
12, named *twelve*, two and ten.
13, named *thirteen*, three and ten.
14, named *fourteen*, four and ten.
15, named *fifteen*, five and ten.
16, named *sixteen*, six and ten.
17, named *seventeen*, seven and ten.
18, named *eighteen*, eight and ten.
19, named *nineteen*, nine and ten.
20, named *twenty*, twice ten (*ty* means *ten*).

EXERCISES.

1. To be named:

1.	11,	15,	20,	13.	6.	16,	13,	12,	18.
2.	19,	16,	14,	19.	7.	20,	17,	16,	13.
3.	12,	17,	11,	15.	8.	11,	14,	13,	19.
4.	18,	15,	19,	16.	9.	18,	16,	17,	14.
5.	13,	18,	15,	17.	10.	12,	15,	12,	20.

2. To be expressed in figures:

1.	Ten,	eighteen,	eleven,	fourteen.
2.	Eleven,	nineteen,	seventeen,	eleven.
3.	Twelve,	fifteen,	twenty,	eighteen.
4.	Thirteen,	twelve,	sixteen,	twenty.
5.	Fourteen,	twelve,	nineteen,	seventeen.

Each of the above numbers consists of two figures: the figure on the right expresses *units;* the figure on the left, *tens.*

3. Point out the number of tens and units in 11, 17, 19, 15, 20.

Numbers from Twenty to One Hundred.

The numbers from one to one hundred form ten groups. The first two groups we have shown you.

Third group: 21, 22, 23, etc., named twenty-one, twenty-two, etc.

Fourth group: 31, 32, 33, etc., named thirty-one, thirty-two, etc.

Fifth group: 41, 42, 43, etc., named forty-one, forty-two, etc.

Sixth group: 51, 52, 53, etc., named fifty-one, fifty-two, etc.

Seventh group: 61, 62, 63, etc., named sixty-one, sixty-two, etc.

Eighth group: 71, 72, 73, etc., named seventy-one, seventy-two, etc.

Ninth group: 81, 82, 83, etc., named eighty-one, eighty-two, etc.

Tenth group: 91, 92, 93, etc., named ninety-one, ninety-two, etc.

NOTE.—The last number of the tenth group is called *one hundred* (100).

EXERCISES.

1. Write in figures, and read, the ten numbers of the third group.

2. Write in figures, and read, the numbers belonging to each of the remaining groups.

3. Read each of the following numbers, and name the tens and units in each:

	(A.)	(B.)	(C.)	(D.)	(E.)
1.	64,	45,	42,	67,	49.
2.	73,	93,	65,	43,	53.
3.	92,	82,	87,	79,	19.
4.	18,	67,	98,	32,	24.
5.	30,	60,	50,	91,	86.
6.	75,	23,	48,	28,	27.
7.	26,	78,	53,	17,	60.
8.	38,	56,	40,	65,	67.
9.	44,	29,	77,	25,	72.
10.	20,	90,	80,	68,	83.

4. Express in figures the following:

	(A.)	(B.)	(C.)
1.	One unit.	Forty-six.	Sixty.
2.	Two units.	Seventy-five.	Thirty-eight.
3.	One ten and three units.	Fifty-three.	Twenty-seven.
4.	Two tens and five units.	Thirty-four.	Seventy-six.
5.	Six tens and five units.	Sixty-nine.	Sixty-eight.
6.	Eight tens and three units.	Ninety-seven.	Thirty-two.
7.	Seven tens and six units.	Eighty.	Eighty-nine.
8.	Two tens and nine units.	Twenty-three.	Thirty-five.

	(A.)	(B.)	(C.)
9.	Eight tens and seven units.	Eighty-one.	Ninety-four.
10.	Nine tens and no units.	Seventy-two.	Forty-one.

5. What does the *ty* in twenty, thirty, etc., mean?

6. Does seventy mean seven and ten, or seven tens?

7. Write and read all the numbers of the first group.

8. Write and read all the numbers of the second group. *20 equals how many tens?*

9. Write and read all the numbers of the tenth group.

10. How many groups of ten numbers each have to be written before reaching one hundred (100)?

11. Then *100 equals how many tens? 10 equals how many units?*

Numbers from 100 to 1000.

1. Since one hundred is written 100,
two hundred is written 200,
three hundred is written 300,
four hundred is written 400,
five hundred is written 500,
six hundred is written 600,
seven hundred is written 700,
eight hundred is written 800,
nine hundred is written 900.

Nine is written thus, 9,
9 units.

Ninety-nine is written thus, 99,
9 tens and 9 units.

Nine hundred ninety-nine is written thus, 999,
9 hundreds, 9 tens, and 9 units.

PRINCIPLE.

A figure standing alone, or in the right-hand place, expresses units; in the second place, tens; in the third place, hundreds.

2. Analysis (Greek, *taking apart*) points out the parts of which a number is composed.

EXERCISES.

1. Analyze and read the number 473. To apply the principle: 4 standing in the third place expresses *hundreds*, 7 standing in the second place expresses *tens*, 3 in the first place expresses *units*. Hence, the number is read *four hundred seventy-three*. Do not read thus: Four hundred *and* seventy-three. The importance of dropping *and* will appear later.

2. In like manner analyze and read the following:

	(A.)	(B.)	(C.)	(D.)	(E.)
1.	613,	702,	962,	814,	104.
2.	724,	359,	483,	902,	134.
3.	538,	400,	965,	893,	246.
4.	904,	916,	684,	246,	408.
5.	523,	820,	793,	489,	325.
6.	186,	547,	924,	249,	767.
7.	248,	962,	998,	568,	482.
8.	517,	483,	594,	346,	618.
9.	342,	965,	489,	200,	821.
10.	981,	684,	993,	108,	604.

3. Write in Arabic characters the following:

1. Three hundred sixteen.
2. Two hundred forty-seven.
3. Three hundred eighty-five.
4. Four hundred nine.
5. Two hundred thirty-five.
6. Six hundred eighteen.
7. Eight hundred forty-two.
8. Seven hundred fifteen.
9. Two hundred forty-three.
10. Eight hundred nineteen.
11. Two hundred seven.
12. Five hundred thirty-nine.
13. Four hundred.
14. Six hundred nineteen.
15. Eight hundred twenty.
16. Four hundred fifty-seven.
17. Two hundred sixty-nine.
18. Three hundred eighty-four.
19. Five hundred sixty-nine.
20. Four hundred eighty-six.
21. Four tens, three units.
22. One hundred, no tens, four units.
23. Two hundreds, four tens, six units.
24. Five hundreds, six tens, eight units.

PERIODS AND ORDERS.

The hundreds, tens, and units composing a number constitute what is called a **Period**, and the three places are called **Orders**.

The figure in units' place denotes units of the *first order;* the figure in tens' place, units of the *second order;* the figure in hundreds' place, units of the *third order.*

EXERCISES.

1. Compose numbers, using the following units and orders:

1. One unit of the first order, two units of the second order, three of the third.
2. Four units of the first order, five of the second, six of the third.
3. Seven of the first order, eight of the second, nine of the third.
4. None of the first, two of the second, three of the third.
5. Four of the second, five of the first, six of the third.
6. Seven of the third, eight of the second, nine of the first.

Suggestion: When an order is not mentioned a cipher must be written in its place.

7. One unit of the first order, two units of the third order.
8. Eight units of the third order, one of the first.
9. Five of the third and three of the second.
10. Nine units of the third order.

2. Having written the above numbers, read each one of them, omitting *and.*

We have now gone far enough to make obvious the following

PRINCIPLES.

1. Ten units of any order make one unit of the next higher order.

2. The removal of a figure one place to the left multiplies its value by ten.

3. The removal of a figure one place to the right divides its value by ten.

4. 0 is used to give place and value to the significant figures.

NOTE.—Let the pupil use a digit to illustrate the above principles.

3. The number next above 999 is called *one thousand* (1000), the 1 being a unit of the fourth order.

The fourth, fifth, and sixth orders complete a second period, called the period of *thousands.*

The seventh, eighth, and ninth orders form the period of *millions.*

The tenth, eleventh, and twelfth orders form the period of *billions.* Following billions are the periods of *trillions*, *quadrillions*, *quintillions*, etc.

4. Each period has units, tens, and hundreds of its own.

5. Since ten units make one ten, and ten tens make one hundred, and ten hundreds make one thousand, and so on, the Arabic system of notation is called the **decimal system.** (Latin, *decem*, *ten.*)

6. The decimal system of notation is best set forth by means of a table.

Table.

Periods.	6th.			5th.			4th.			3d.			2d.			1st.		
Names.	Quadrillions.			Trillions.			Billions.			Millions.			Thousands.			Units.		
Name of Orders.	Hundreds of Quadrillions.	Tens of Quadrillions.	Quadrillions.	Hundreds of Trillions.	Tens of Trillions.	Trillions.	Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
Number.	5	6,		9	2	0,	7	4	1,	6	5	8,	3	2	4,	5	0	6

This number, as analyzed by the table, is to be read thus: *Fifty-six* quadrillion, *nine hundred twenty* trillion, *seven hundred forty-one* billion, *six hundred fifty-eight* million, *three hundred twenty-four* thousand, *five hundred six.*

7. Notice the following features of the number in the table:

1. The periods of the number are set off by commas.
2. All the periods have three digits except the sixth.
3. In reading, the name of the first period (units) is omitted.
4. The absence of significant figures is indicated by 0.

Note.—A whole period may be indicated by three ciphers.

8. Name the following periods:

1.	First.	Sixth.	Third.	Sixth.	First.
2.	Second.	First.	Fourth.	Fifth.	Second.
3.	Third.	Fifth.	Second.	Fourth.	Third.
4.	Fourth.	Second.	Fifth.	Third.	Fourth.

9. Give the number of the following periods:

1.	Units.	Quadrillions.	Billions.
2.	Thousands.	Trillions.	Units.
3.	Millions.	Billions.	Thousands.
4.	Billions.	Millions.	Trillions.
5.	Trillions.	Thousands.	Millions.

10. Name the following orders:

1.	First.	Second.	Fourth.	Thirteenth.
2.	Fourth.	Third.	First.	Sixteenth.
3.	Seventh.	Sixteenth.	Second.	First.
4.	Tenth.	First.	Third.	Fourth.
5.	Second.	Thirteenth.	Fourth.	Seventh.
6.	Thirteenth.	Tenth.	Seventh.	Tenth.

NOTE.—Observe that the first, fourth, seventh, tenth, thirteenth, and sixteenth orders name the periods to which they respectively belong.

11. Give the names of the following periods and orders:

	Periods.	Orders.		Periods.	Orders.
1.	6th	16th.	9.	5th	15th.
2.	5th	13th.	10.	4th	11th.
3.	4th	10th.	11.	4th	12th.
4.	3d	7th.	12.	3d	8th.
5.	2d	4th.	13.	3d	9th.
6.	1st	1st.	14.	2d	5th.
7.	6th	17th.	15.	2d	6th.
8.	6th	18th.	16.	1st	2d.

12. Copy carefully and point off into periods:

1. 5056.	11. 7385062.	21. 46825.
2. 5565.	12. 8237185.	22. 25665.
3. 6065.	13. 3785024.	23. 54646.
4. 46632.	14. 6083219.	24. 356236.
5. 64645.	15. 7882709.	25. 2653665.
6. 66646.	16. 38420058.	26. 71234567.
7. 54532.	17. 33468204.	27. 589012345.
8. 654653.	18. 5284325684.	28. 5678010112.
9. 466214.	19. 7932468412.	29. 61314151617.
10. 664536.	20. 83749275867.	30. 618192021222.

31. 655256234625499844.

Analyze and read the first number, 5056.

Process. **Analysis.**

5,056 Pointing off, we find there are two periods: thousands and units;—in the higher period, 5 thousands; in the other, 0 hundreds, 5 tens, 6 units. Hence, the number is read *Five thousand fifty-six.*

Analyze and read the eleventh number, 7385062.

Process. **Analysis.**

7,385,062 Pointing off, we find there are three periods: millions, thousands, units. Beginning with the highest period and reading, we have *Seven million three hundred eighty-five thousand sixty-two.*

In like manner analyze and read each of the numbers in the first column.

Brief directions are:

1. **Copy the number.**
2. **Beginning at the right, point off.**
3. **Beginning at the left, read.**
4. **Omit the last name, units.**

13. Apply the directions to the following:

1. 2725.	15. 82638.	29. 7904804.
2. 7637.	16. 57188.	30. 9265418.
3. 2754.	17. 88765.	31. 575306.
4. 7237.	18. 38577.	32. 820583.
5. 6675.	19. 82683.	33. 5457474.
6. 4667.	20. 87683.	34. 50767576.
7. 8263.	21. 7385062.	35. 756272376.
8. 8878.	22. 8237185.	36. 3838785838.
9. 7392.	23. 3785024.	37. 46887758381.
10. 7940.	24. 6083219.	38. 500688000233.
11. 75324.	25. 7882709.	39. 8638866804000.
12. 61764.	26. 8563988.	40. 97000001543210.
13. 75773.	27. 6473978.	41. 55555555555555.
14. 57267.	28. 5656300.	42. 98765432101234.

14. Write in Arabic numerals thirty-six million twenty-nine thousand fifty.

Process.

3 2 1
m. th. u.
. . . , . . . , . . .
36,029,050

Explanation.

Since there are three periods,—*millions*, *thousands*, *units*,—we write three dots for each to indicate the number of places to be filled. We place 36 in the third period, 29 in the second, and 50 in the first. The places remaining unoccupied we fill with ciphers, and have 36,029,050.

Brief directions are:

1. Ascertain the number of periods and orders.

2. Beginning at the left, write the significant figures of each period, filling vacant places with ciphers.

15. Apply the rule to the following:

1. One thousand two hundred thirty-four.
2. Five thousand six hundred seventy-eight.
3. Nine thousand twelve.
4. Three thousand four hundred fifty-six.
5. Seven thousand eight hundred ninety.
6. One thousand two hundred thirty-four.
7. Nine thousand one.
8. Three thousand three hundred forty-four.
9. Nine thousand nine hundred.
10. Five thousand five.
11. Eight million one thousand seventy-six.
12. Three million one hundred sixty-two thousand one hundred seventy-two.
13. Forty million twenty-seven thousand six hundred twenty-one.
14. One hundred twenty-three million four hundred fifty-six thousand seven hundred eighty-nine.
15. Nine hundred eighty-seven million six hundred fifty-four thousand three hundred twenty-one.
16. Two hundred thirty-one million two hundred two thousand seven hundred.
17. Thirty million eight hundred twenty-six thousand fifty-one.
18. Seven million eight hundred one thousand seventeen.
19. Four hundred fifty-eight million two hundred seventy-five thousand six hundred ten.
20. Ninety million five hundred seventy-seven thousand six hundred nineteen.
21. Twenty-nine billion one million one thousand eight hundred.

22. Fifty-nine billion one million one.

23. Six hundred twenty million eighty-four thousand.

24. Four hundred sixty-seven million nine thousand nine.

25. Fifty billion fifty million fifty thousand fifty.

26. Six hundred billion six hundred million six hundred.

27. Forty-three billion twenty million twenty-four thousand.

28. Eighty-six billion eighty-four thousand five.

29. Five hundred twenty-five million five.

30. Three hundred twenty-five billion seventeen million ninety thousand nine hundred ninety.

31. Six hundred twenty-five million five hundred thousand twenty-five.

32. Seven hundred twenty-six million eight thousand eight hundred eighty.

33. Three hundred twenty-nine thousand three hundred three.

34. Three hundred million two hundred twenty-seven.

35. Sixty-six billion one hundred million one hundred seventy-four thousand seventy-four.

36. Eighty million eighty thousand eight.

37. Ninety-five billion seven million six thousand one hundred seventy-five.

38. Seventy-six million twenty-four.

39. Nine million twenty-eight thousand.

40. One million.

41. Three million, ten.

42. Seventy-six million twenty-four.

43. Four hundred million three hundred twenty-nine.

44. Fifty-five billion two hundred million nine hundred eighty-four thousand seventy-five.

45. Seventeen million four hundred seven thousand eighty-four.

46. Twenty-eight million five hundred ninety-four thousand sixty-seven.

47. Eighty million eighty thousand eight.

48. Ninety-five billion nine million eight thousand two hundred ninety-four.

49. Six hundred fifty-five quadrillion two hundred fifty-six trillion two hundred thirty-four billion six hundred twenty-five million four hundred ninety-nine thousand eight hundred forty-four.

DECIMAL NOTATION.

1. The **Decimal System** of notation extends to the decimal parts of a unit, called *tenths, hundredths, thousandths.*

Decimal System.

Ascending Scale.				*Descending Scale.*		
10 hundreds,	10 tens,	10 units,	Unit	$\frac{1}{10}$ of a unit,	$\frac{1}{10}$ of a tenth,	$\frac{1}{10}$ of a hundredth,
1000	100	10	1	.1	.01	.001
one thousand.	one hundred.	one ten.	ONE	one tenth.	one hundredth.	one thousandth.

Reading by columns from the unit towards the left, we have the decimal system of whole numbers; reading by columns from the unit towards the right, we have the decimal system of fractions.

2. To mark the beginning of tenths and to separate tenths and units a sign [.] called the **Decimal Point** is used.

4.235 is read, "4 units *and* 2 tenths, 3 hundredths, 5 thousandths"; or, more briefly, "4, *and* 235 thousandths," the decimal name of the last figure only being repeated.

Notice here that the decimal point is read *and*. Moreover, that in reading numbers *and* is to be applied to the decimal point exclusively.

3. When the decimal point simply marks the beginning of a number it is not read. .67 is read *sixty-seven hundredths*.

.5 and .50 are of the same value; but .05 is one-tenth the value of .5. [See Principle 3, page 11.]

EXERCISES.

1. Read the following decimals:

1. .6.	11. .06.	21. 3.38.	31. .303.
2. .9.	12. .09.	22. 4.57.	32. 300.007.
3. .13.	13. .013.	23. 5.066.	33. 1728.144.
4. .75.	14. .075.	24. 6.842.	34. 2150.4.
5. .96.	15. .020.	25. 17.825.	35. 3.141.
6. .548.	16. .01.	26. 30.054.	36. 365.25.
7. .636.	17. .001.	27. 33.005.	37. .003.
8. .045.	18. .80.	28. 215.455.	38. .300.
9. .008.	19. .500.	29. 326.008.	39. 4444.444.
10. .023.	20. .625.	30. 400.000.	40. 5000.005.

2. Write in figures:
   1. Four tenths. Eight tenths. Nine tenths.
   2. Six hundredths. Fourteen hundredths. Twenty-one hundredths.

3. Seventeen thousandths. Three hundred three thousandths.
4. Eighty-five hundredths. Eight, *and* five tenths.
5. Sixty, *and* six hundredths. Five, *and* fifteen thousandths.
6. Nine, *and* two hundred sixteen thousandths.
7. Seventy-three, *and* eight tenths. One tenth. One hundredth.
8. One thousandth. Two hundredths. Three tenths.
9. One thousand, *and* two hundredths. Three hundred, *and* three tenths.
10. Two hundred sixteen, *and* three hundred eighty-four thousandths.

3. Does annexing a cipher to a decimal alter its value?

4. What is the effect of inserting a cipher between the decimal point and the first figure of the decimal?

5. What is the effect of moving decimal figures towards the right?

UNITED STATES MONEY.

1. The decimal system of notation and numeration applies to United States currency, in which

10 mills make 1 cent.
10 cents make 1 dime.
10 dimes make 1 dollar.
10 dollars make 1 eagle.

2. Dollars are denoted by the sign $ written before the number. $12 means twelve dollars.

3. The decimal point is placed between dollars and dimes; hence, dimes are *tenths* of a dollar, cents are *hundredths* of a dollar, and mills *thousandths* of a dollar.

In practice the word *dimes* is not much used; $1.25 is commonly read, One dollar and twenty-five cents. It may also be read, One dollar and twenty-five hundredths.

EXERCISES.

1. Read 5.875 and give each figure its appropriate name.

2. Read $5.875 and give each figure its appropriate name.

3. Read $5.875 as dollars, cents, and mills.

4. Read the following:

1. $ .15.	10. $ 16.540.	19. $329.357.
2. $ 2.25.	11. $ .57.	20. $295.403.
3. $ 5.375.	12. $ .625.	21. $ 20.21.
4. $ 1.060.	13. $ .375.	22. $ 1.011.
5. $18.100.	14. $ 3.94.	23. $ .10.
6. $35.27.	15. $ 3.56.	24. $ .05.
7. $46.39.	16. $429.384.	25. $ .01.
8. $ 3.368.	17. $325.495.	26. $ 4.875.
9. $ 2.075.	18. $426.384.	27. $ 16.075.

5. Write the following:

1. Seven dollars, forty-nine cents.
2. Fourteen dollars, nine cents.
3. Seventy dollars, twenty-five cents.
4. Five dollars, forty-one cents, five mills.
5. Eighty-seven cents, five mills.
6. Ninety dollars, seven cents.
7. Twenty-seven dollars, fifty-six cents.
8. Thirty-seven dollars, five dimes.
9. Nine dollars, thirty-three cents, three mills.
10. Twenty-seven cents, eight mills.

11. Eighty-six dollars, five cents, two mills.
12. Twenty-one dollars, twenty-one cents, one mill.
13. Thirty-seven dollars, eighteen cents, eight mills.
14. Forty-nine dollars, nine cents, six mills.
15. One hundred five dollars, seventy cents.
16. Twenty-seven dollars, *and* four tenths.
17. Three thousand dollars, *and* fifty hundredths.
18. Ten dollars, ten cents, two mills.
19. Twenty-five dollars, three dimes, seven cents, five mills.
20. One dollar, one cent, one mill.
21. Twenty dollars, twenty cents, *ten mills.*

ROMAN NOTATION.

The **Roman System** of notation expresses numbers by means of seven capital letters, viz.:

Letters: I., V., X., L., C., D., M.
Values: 1, 5, 10, 50, 100, 500, 1000.

To express other numbers these letters are combined:

Table.

I. 1	XIV. 14	LX. 60
II. 2	XV. 15	LXX. 70
III. 3	XVI. 16	LXXX. . . . 80
IV. 4	XVII. 17	XC. 90
V. 5	XVIII. 18	C. 100
VI. 6	XIX. 19	CC. 200
VII. 7	XX. 20	CCL. 250
VIII. 8	XXI. 21	CCCC. 400
IX. 9	XXIX. 29	D. 500
X. 10	XXX. 30	DCC. 700
XI. 11	XXXIV. 34	M. 1000
XII. 12	XL. 40	MMM. 3000
XIII. 13	L. 50	$\overline{\text{XVI}}$. . . . 16000

The table shows that combinations are made:

1. By repeating any of the letters, except V., D., and L., as II., XX.
2. By writing a letter of less value *after* one of greater value, as VI., XV.
3. By writing a letter of less value, except V. and D., *before* one of greater value, as IX., XL.
4. By writing a letter of less value *between* two of greater value, as XIV.

The letter standing before an inserted letter cannot be *less in value* than the letter following it: XIV., not VIX.

5. By placing a bar over a letter or a combination of letters, as $\overline{V}$ or $\overline{XI}$.

The effect of these combinations may be stated briefly as follows:

PRINCIPLES.

1. **Repeating a letter repeats value, as in XX.**
2. **Affixing a letter increases value, as in XI.**
3. **Prefixing a letter diminishes value, as in IX.**
4. **Inserting a letter diminishes value, as in XIX.**
5. **Placing a bar increases value, as in $\overline{VI}$.**

EXERCISES.

1. Let the principles answer the following questions:
   1. What does the combination III. express? Why?
   2. What does the combination IV. express? Why?

3. What does the combination XV. express? Why?
4. What does the combination XIV. express? Why?
5. What does VIX. express? Why?
6. How is 14 properly expressed?
7. What is the value of MIX.? Why?
8. What is the value of LIX.? Why?
9. What is the value of IXL.? Why?
10. What is the value of LXI.? Why?
11. Is it a letter of less value or of greater value that is prefixed, affixed, or inserted?
12. Can a letter before an inserted letter be less than the one following it?

2. Read the following:

1. XX.	17. LXIX.	33. XCI.
2. XXI.	18. IXIV.	34. DCXC.
3. XXIV.	19. XC.	35. DCCX.
4. XXV.	20. XCIV.	36. XXXIX.
5. XXVI.	21. XCIX.	37. LXXXIX.
6. XXVIII.	22. XLIV.	38. CLXXIX.
7. XIX.	23. CDXX.	39. CCCCXIVII.
8. XXXII.	24. CCXXIV.	40. VIII.
9. XLV.	25. VIII.	41. CCXC.
10. LXV.	26. XXIX.	42. CXLIX.
11. XXXV.	27. MDLIV.	43. CLI.
12. XL.	28. XXXIII.	44. MD.
13. LX.	29. CIX.	45. CIX.
14. LXX.	30. CXI.	46. LXV.
15. LVIII.	31. CXLV.	47. $\overline{M}$.
16. LXXX.	32. CCXIX.	48. $\overline{XIV}$.

3. Express in Roman notation:

1. 25.	11. 58.	21. 387.	31. 25.	41. 100.
2. 45.	12. 97.	22. 587.	32. 46.	42. 2000.
3. 16.	13. 324.	23. 436.	33. 52.	43. 2505.
4. 38.	14. 423.	24. 789.	34. 87.	44. 3333.
5. 72.	15. 520.	25. 207.	35. 77.	45. 4444.
6. 47.	16. 337.	26. 999.	36. 66.	46. 5555.
7. 95.	17. 495.	27. 1000.	37. 56.	47. 6666.
8. 69.	18. 327.	28. 1500.	38. 51.	48. 7777.
9. 73.	19. 514.	29. 1550.	39. 49.	49. 8888.
10. 46.	20. 599.	30. 1555.	40. 99.	50. 1892.

ADDITION.

Oral.

INDUCTIVE STEPS.

How many are:

1. 2 cents and 2 cents? 2 and 2?
2. 3 pencils and 2 pencils? 3 and 2?
3. 4 doors and 3 doors? 4 and 3?
4. 5 windows and 3 windows? 5 and 3?
5. 6 birds and 4 birds? 6 and 4?
6. 4 birds and 6 birds? 4 and 6?
7. 5 fingers and 5 fingers? 5 and 5?
8. 7 fields and 3 fields? 7 and 3?
9. 7 fields and 3 *trees?*

Since 7 and 3 are ten, why cannot you say that 7 fields and 3 trees are 10? What is the unit of each number?

Then we must conclude that only numbers having the same unit can be expressed in a single number.

DEFINITIONS.

1. Numbers having units of the same kind are called **Like Numbers.**

2. **Addition** is the process of uniting two or more numbers into a single number, called their **Sum.**

3. The sum, then, is the result obtained by adding.

4. The **Sign of Addition** is an upright cross (+) called **plus** (more); it is placed between the numbers to be added. $7 + 4$ is read "7 plus 4."

5. The **Sign of Equality** is two parallel horizontal lines, =. $7 + 4 = 11$ is read "7 plus 4 equals 11."

6. $7 + 4 = 11$, being an expression of **equality**, is called an **Equation.**

PRINCIPLES.

1. **Only like numbers can be added.**

2. **The sum and the numbers added are like numbers.**

EXERCISES.

1. The digits are: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. What numbers do they represent?

2. Answer the following questions at sight:

1. $0 + 1 = ?$	8. $7 + 0 = ?$	15. $4 + 5 = ?$
2. $1 + 3 = ?$	9. $8 + 1 = ?$	16. $5 + 6 = ?$
3. $2 + 2 = ?$	10. $9 + 2 = ?$	17. $6 + 4 = ?$
4. $3 + 1 = ?$	11. $0 + 4 = ?$	18. $7 + 5 = ?$
5. $4 + 3 = ?$	12. $1 + 5 = ?$	19. $8 + 6 = ?$
6. $5 + 1 = ?$	13. $2 + 6 = ?$	20. $9 + 4 = ?$
7. $6 + 2 = ?$	14. $3 + 4 = ?$	21. $0 + 7 = ?$

22. $1 + 8 = ?$	29. $8 + 9 = ?$	36. $5 + 5 = ?$
23. $2 + 9 = ?$	30. $9 + 7 = ?$	37. $6 + 6 = ?$
24. $3 + 7 = ?$	31. $0 + 0 = ?$	38. $7 + 7 = ?$
25. $4 + 8 = ?$	32. $1 + 1 = ?$	39. $8 + 8 = ?$
26. $5 + 9 = ?$	33. $2 + 2 = ?$	40. $9 + 9 = ?$
27. $6 + 7 = ?$	34. $3 + 3 = ?$	41. $9 + 2 = ?$
28. $7 + 8 = ?$	35. $4 + 4 = ?$	42. $9 + 6 = ?$

(A.)	(B.)	(C.)
1. 3, 2, 5.	6. 3, 4, 2, 5.	11. 5, 4, 3, 7.
2. 4, 3, 2.	7. 2, 3, 4, 6.	12. 6, 8, 4, 3.
3. 5, 4, 3.	8. 6, 3, 2, 4.	13. 2, 9, 6, 2.
4. 6, 2, 4.	9. 5, 7, 3, 6.	14. 4, 6, 8, 3.
5. 5, 3, 2.	10. 8, 4, 2, 3.	15. 7, 8, 4, 9.

3. Begin at the uppermost 3 in A, and give orally and at sight the sum of the numbers in each line thus: 3, 5, *10*, sum of first line of A.

4. Begin at the same place and give in the same manner the sum of each column thus: 3, 7, 12, 18, *23*, the sum of the first column of A.

ORAL PROBLEMS.

1. How many fingers are 6 fingers and 4 fingers?

Answer: 6 fingers + 4 fingers = 10 fingers.

2. If an apple cost 3 cents and an orange 5 cents, how much did both cost?

3. A girl paid 6 cents for a pencil and 5 cents for sugar-plums. How much did she pay for both?

4. A boy spent 7 cents for some paper and 5 cents for an orange. How much did he spend for both?

5. A man rode on his bicycle 8 miles the first hour

and 7 miles the second hour. How far did he ride in the two hours?

6. A horse travels 9 miles the first hour and 8 miles the second hour. How far does he go in two hours?

7. A farmer who had 5 horses, bought 5 more at one time, and at another time 4 more. How many did he then have?

8. A matron bought 8 quarts of strawberries of one man, 5 quarts of another, and 5 of another. How many quarts did she buy in all?

9. A traveller bought for lunch 5 cents' worth of crackers, 4 cents' worth of milk, and 8 cents' worth of berries. What did his lunch cost him?

10. How many *animals* are in a field, there being 6 cows, 4 oxen, and 8 sheep?

11. Margaret gave $2.00 for an atlas, $3.00 for a history, and $8.00 for a dictionary. How much did she pay for all?

12. A man earns $9.00, while a boy and a girl earn each $5.00. How much do all earn when the man has earned $9.00?

13. Add 2, 6, 8, 7, 1, 2. Also, 8, 5, 3, 2, 4, 9.

14. A house has 8 windows on the east side, 7 on the west, and 9 on the south. How many are there in all?

15. A rose-bush has on one branch 8 roses, on another 7 roses, and on a third 6 roses. How many roses on the three branches?

16. In one part of a building there are 9 offices, in another part 5 offices, in a third part 6 offices. How many in the three parts?

17. Jennie sold to a lady 4 quarts of berries, 8 quarts

of peaches, 2 quarts of beans, and 1 quart of peas. How many quarts did the lady buy?

18. A gentleman selected for a bouquet 8 white carnations, 6 pink ones, and 6 red ones. How many in all in the bouquet?

19. Sarah wrote on Monday 9 letters, on Tuesday 8 letters, on Wednesday 7 letters. How many letters did she write in the three days?

20. A foot-ball team won 2 games in September, 3 games in October, 4 games in November, not counting the game won on Thanksgiving Day. How many games were won in the three months?

21. Count by 2's from 0 to 50; thus: 0, 2, 4, 6, etc.

22. Count by 3's from 0 to 51. From 4 to 47.

23. Count by 4's from 0 to 52. From 5 to 59.

24. Count by 6's from 0 to 72. From 5 to 77.

25. Count by 7's from 0 to 84. From 6 to 90.

26. Count by 8's from 0 to 96. From 4 to 100.

37. Count by 9's from 7 to 115. From 0 to 117.

Addition of Single Columns.

WRITTEN EXERCISES.

1. What is the sum of 3, 6, 8, 7?

Process.	Explanation.
3 6 8 7 24	Explain, speaking thus: "For convenience, we write the numbers in a column. We then add thus: $7 + 8 = 15$; $15 + 6 = 21$; $21 + 3 = 24$, the sum." To prove the correctness of the result, we add the column downwards: $3 + 6 = 9$; $9 + 8 = 17$; $17 + 7 = 24$, the sum. We may explain more briefly, saying "3, 9, 17, *24*; again, 7, 15, 21, *24*."

What is the first principle of Addition?
Are the preceding added numbers like numbers?
They are all units of what order?

2. In a similar manner add and explain these:

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)	(9.)	(10.)
7	5	9	4	8	7	4	5	3	9
5	7	3	7	9	2	8	9	7	8
8	6	8	3	6	5	3	1	8	2
4	4	4	5	5	6	6	6	6	5

(11.)	(12.)	(13.)	(14.)	(15.)	(16.)	(17.)	(18.)	(19.)	(20.)
6	8	6	6	8	3	6	5	2	2
3	5	3	3	7	5	4	9	7	9
5	5	9	1	4	2	4	3	4	6
9	7	9	5	4	7	8	6	4	4

(21.)	(22.)	(23.)	(24.)	(25.)	(26.)	(27.)	(28.)	(29)	(30.)
2	7	3	5	9	6	5	9	4	2
9	8	8	2	7	6	8	7	8	9
6	2	6	6	3	7	7	6	6	8
4	6	1	8	1	2	6	8	2	4
7	5	9	3	5	4	9	3	8	5

(31.)	(32.)	(33.)	(34.)	(35.)	(36.)	(37.)	(38.)	(39.)	(40.)
6	7	3	9	5	5	7	6	9	8
9	4	9	6	6	2	3	3	4	7
7	1	2	5	4	8	5	8	7	5
5	6	7	2	7	6	8	6	3	5
8	3	5	8	1	3	6	4	6	3

3. Required the sum of the following numbers:

1. 4, 5, 2, 4, 8, 6.	11. 6, 9, 7, 5, 8, 4.
2. 7, 6, 4, 8, 5, 3.	12. 7, 4, 1, 6, 3, 8.
3. 9, 3, 5, 6, 4, 2.	13. 3, 9, 2, 7, 5, 1.
4. 8, 6, 8, 5, 2, 3.	14. 9, 6, 5, 2, 8, 7.
5. 4, 3, 8, 6, 5, 4.	15. 5, 6, 4, 7, 1, 8.
6. 8, 9, 3, 1, 9, 4.	16. 5, 2, 8, 6, 3, 4.
7. 6, 7, 9, 4, 5, 6.	17. 7, 3, 5, 8, 6, 1.
8. 4, 9, 3, 6, 8, 2.	18. 9, 5, 9, 4, 6, 8.
9. 7, 3, 4, 8, 2, 5.	19. 3, 6, 8, 5, 6, 3.
10. 4, 7, 5, 6, 2, 4.	20. 7, 4, 9, 4, 9, 3.

WRITTEN PROBLEMS.

1. A book-agent sold 3 books every day for six days. How many books did he sell in the six days?

3 + 3 + 3 + 3 + 3 + 3 = number sold.

NOTE.—Let the pupil indicate the solution of each problem in this way.

2. In a family are six children; the youngest is 2 years old. Each of the other five is 2 years older than the one next younger. What is the age of the oldest?

3. A tree was broken off 9 feet from the ground; 8 feet of the piece broken off lies upon the ground, and the remaining 7 feet of it lies in water. How high was the tree?

4. A book-case contained 5 shelves. The librarian removed 9 books from the fifth shelf, 8 books from the fourth, 5 books from the third, 7 books from the second, and 4 books from the first. How many books did he remove from the case?

5. A man on examining his purse found therein 5 dimes, 4 quarter-dollars, 3 dollars (silver), 2 half-dollars, and one eagle. How many pieces of money had he?

6. I have three different measures: gill, pint, quart. The pint contains 4 gills, the quart 8 gills. How many gills do the three measures contain?

7. If a pint of water weighs one pound, and 2 pints equal 1 quart, and 8 pints equal 1 gallon, how much does the water in a pint, a quart, and a gallon measure weigh?

8. John, James, George, and Henry were solving problems. John solved 5, James solved 4, George as many as both John and James. Henry copied the 4 that James did. How many problems were solved?

9. A room is 9 feet long, 9 feet wide, and 8 feet high. How long a line would measure twice across the width of the room and then up to the ceiling?

10. Some articles on my desk measure as follows: A pen-holder, 7 inches; a steel ink-eraser, 7 inches; a pair of dividers, 6 inches; a lead-pencil, 5 inches; a paper-cutter, 8 inches; and a fountain-pen, 7 inches. Find the combined length of the six articles.

11. On a wintry holiday 6 boys and 4 girls, set free from school, went skating; 7 girls and 5 boys went coasting; 4 boys built a snow man, and 2 a snow fort. Find the number of boys, the number of girls, and the number of pupils the school contained.

12. A street-vender sold to some boys 4 apples for 8 cents, 3 oranges for 9 cents, 2 pears for 6 cents, 5 lemons for 9 cents, and 2 peaches for 4 cents. How many units of fruit did he sell, and how many cents did he receive?

13. A school-girl wrote in her copy-book 7 lines on Monday, 8 lines on Tuesday, 6 lines on Wednesday, 5 lines on Thursday, and 7 lines on Friday. How many lines did she write during the week?

14. Find the sum of 1, 2, 3, 4, 5, 6, 7, 8, 9.

15. Change to Arabic numerals the following Roman numbers, and find their sum: III., IX., VIII., V., IV., VI.

ORAL EXERCISES.

1. How many are:
    1. 10 cents and 10 cents? 10 and 10?
    2. 10 dollars and 20 dollars? 10 and 20?
    3. 10 tops and 30 tops? 10 and 30?
    4. 30 books and 10 books? 30 and 10?
    5. 35 books and 10 books? 35 and 10?
    6. 10 books and 40 books? 10 and 40?
    7. 40 men and 10 men? 40 and 10?
    8. 40 men and 8 men? 40 and 8?
    9. 10 men and 18 men? 10 and 18?
    10. 20 ships and 16 ships? 20 and 16?
2. Count by:
    1. 10's from 0 to 100. From 2 to 92.
    2. 10's from 5 to 95. From 4 to 94.
    3. 10's from 7 to 97. From 3 to 93.
    4. 10's from 8 to 98. From 19 to 90.
    5. 10's from 9 to 99. From 1 to 101.
    6. 20's from 0 to 100. From 5 to 85.
    7. 20's from 2 to 102. From 6 to 86.
    8. 20's from 3 to 103. From 7 to 87.
    9. 30's from 5 to 95. From 8 to 98.
    10. 30's from 10 to 100. From 9 to 99.

Addition of Several Columns.

WRITTEN EXERCISES.

1. Find the sum of \$357, \$470, \$534.

Process.	Explanation.
\$357 470 534 \$1361	Explain, speaking thus: "Since the numbers express the same kind of thing, they are *like* numbers, and their sum can be found. In accordance with the same principle, we write units of the same order in the same column, and proceed thus: 4 units + 7 units = 11 units = 1 ten and 1 unit; 1 ten + 3 tens + 7 tens + 5 tens = 16 tens = 1 hundred and 6 tens; 1 hundred + 5 hundred + 4 hundred + 3 hundred = 13 hundred = 1 thousand and 3 hundreds. Hence, the sum is \$1361."

Repeat the principle on which your work depends.

What is a unit? What is the unit of each of the numbers added? What is the unit of their sum?

Can the unit of the numbers added and the unit of their sum ever be different?

Brief directions are:

1. **Write units of the same order in the same column.**
2. **Begin at the right to add.**
3. **Add the columns separately.**
4. **When tens of any order appear, carry them to the next column.**
5. **Write the entire sum of the last column.**
6. **Prove by adding each column in reverse order.**

2. Add and prove each of these:

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
435	340	629	375	390	238	4	310
576	457	308	420	534	354	44	73
357	328	594	965	728	96	444	206

(9.)	(10.)	(11.)	(12.)	(13.)	(14.)
$132	$12.09	$62.22	$2.42	$27.16	$43.38
108	13.17	119.93	6.54	36.59	437.84
159	4.60	7.37	107.31	19.31	394.38
122	4.18	8.04	3.52	45.42	517.73
81	2.58	14.17	6.09	23.08	654.85
106	1.61	13.71	11.56	24.25	826.24

When dollars and cents are written in columns, do not the decimal points form a column?

Did you place a decimal point in each sum directly under the column of points?

3. Add and prove each of these:

(1.) *Soldiers.*	(2.) *Bushels.*	(3.) *Units.*	(4.) *Miles.*
42790	5279	3078	1
42730	570	596	96
4203	4369	9	538
537	520	1034	38
48	210	1005	176
1	22	3333	4444

4. Add 4795, 3084, 3970, 6952, 7964.
5. Add 5968, 3075, 493, 3980, 77.
6. Add 2325, 2642, 5236, 8230, 3616, 21.
7. Add 4836, 658, 816, 636, 1158, 6.
8. Add 52055, 3650, 62055, 4268, 13670, 231.
9. Add 46632, 5056, 64645, 5565, 66646, 54532.
10. Add $75.32, $27.25, $76.37, $27.54, $72.37.
11. Add $617.64, $763.55, $377.07, $767.76, $735.67.
12. Add $767.37, $54.76, $67.54, $476.71, $32.55.

13. Add $67.645, $75.464, $57.677, $74.766, $67.257.
14. Add $82.638, $82.63, $571.88, $88.765, $7.87.
15. Add $472.49, $57.912, $7.392, $93.494, $79.40.
16. Add the following:
    1. Fifty-four thousand, two thousand sixty-five.
    2. Nine thousand, six thousand seventy-one.
    3. Eighteen thousand, six thousand four.
    4. Thirty-five thousand, four hundred thousand.
    5. Two million, three million, four thousand, 867.
    6. Eight million, five hundred thirty-nine million.
    7. Twenty-one million, eight million, 788.
    8. Four hundred, twenty-nine thousand, 500.
    9. Nine million, eight hundred, five thousand, 639.
    10. Eight hundred, twenty-one thousand, forty-seven.
17. Find the sum of the following:
    1. Twenty-seven dollars, fifty-six cents; ninety dollars, twenty cents; seventy-five dollars, twenty-seven cents; nine hundred twenty-seven dollars, eighty cents; thirty-nine dollars, fifty cents.
    2. Three hundred dollars, nine cents; twenty-nine dollars, seven cents; five dollars, forty-one cents, five mills; forty dollars, seventy-nine cents; seven dollars, sixty-two cents, five mills.
    3. Four hundred twenty-nine dollars, seventy cents; seventy dollars, twenty-five cents; eighty-seven cents, five mills; one hundred sixty-nine dollars, twenty-eight cents; thirty-three dollars, sixty-seven cents, five mills.

4. Five hundred sixty dollars, eighty-two cents; nine hundred eighty-seven dollars; ninety dollars twenty cents.

5. Six hundred dollars, eighty-nine cents; one hundred dollars, forty-seven cents; two hundred thirty-four dollars, eighty cents; five hundred fifty-four dollars, ninety cents; nine dollars, thirty-one cents, two mills five-tenths of a mill.

WRITTEN PROBLEMS.

1. A man owns $7580 in land, $4750 in live stock, $1675 in notes, and $2987 in cash. How much is he worth?

NOTE.—Indicate all processes, using proper signs; thus, $7580 + $4750 + $1675 + $2987 = Amount.

2. At the battle of Gettysburg the loss in the Union army was 2,834 men killed and 13,790 wounded; and in the Confederate army, 4,500 killed and 26,500 wounded. What was the entire loss in both armies?

3. A man paid $375 for a carriage, $250 for a horse, and $175 for harness. How much did he pay for all?

4. A grocer bought some sugar for 8 dollars and some tea for 7 dollars. What amount will he receive for the two that he may gain 6 dollars?

5. Suppose a merchant has 3756 dollars in bank-bills, 4793 dollars in gold, 264 dollars in silver, and 5 dollars in cents. How much money has he?

6. The distance from A. to B. is 370 miles, from B. to C. 465 miles, from C. to D. 329 miles. How far is it from A. to D.?

7. I received $53.70 for 105 bushels of oats and $73.20 for 112 bushels of corn. How much money did I receive? How many bushels did I sell?

8. In 1870 the population of New York City was 942,292; of Philadelphia, 674,022; of Brooklyn, 396,099; of St. Louis, 310,864; and of Chicago, 298,977. What was the total population of those cities?

9. I paid away $396 in gold, $574 in silver, and $413 in notes, and had $413 left. How many dollars had I at first?

10. A man making his will left $3450 to his wife, $2675 to his oldest son, $1850 to his second son, and $1290 to his youngest son. What amount of money was bequeathed in his will?

11. Abraham lived 175 years; Isaac, 180; Jacob, 147; Joseph, 110; Moses, 120; Joshua, 110. Find the sum of their ages.

12. A coal-dealer furnished me 8 loads of coal, weighing as follows: 2120 pounds, 2312 pounds, 2218 pounds, 1927 pounds, 2063 pounds, 2284 pounds, 1995 pounds, 1987 pounds. What was the weight of all?

13. B. paid $650 for a lot, on which he built a house for $5785. The fence around the lot cost $167, the pavement cost $243, and the plumbing cost $315. Selling the property, he gained $1200. How much did he receive for it?

14. I bought 40 pounds of sugar for 187 cents, 10 pounds of coffee for 273 cents, and 75 pounds of rice for 357 cents. How many cents did I give for all? How many dollars and cents? How many pounds did I buy?

15. Add:

(1.)	(2.)	(3.)	(4.)	(5.)
923	436	9945	5967	LXXX.
548	597	6878	4304	XC.
796	435	5904	5706	C.
837	608	9237	9708	CC.
537	957	9705	9543	CCL.
943	890	6342	5556	D.
324	457	4356	4395	XIV.
534	565	5274	4950	XVI.
798	402	9252	3076	XIX.
932	917	6244	2705	XXIX.
456	393	4270	8394	XXXIV.
539	407	9396	7657	XLVII.
798	039	6250	3203	LXXXV.
630	575	4396	4308	MMM.
235	970	5394	9867	DLXXX.
497	875	4379	9549	LXXVI.
538	579	6830	6870	DCCC.
925	689	5364	4396	XM.
989	772	8458	9244	MXC.
922	685	7793	8859	MDCCCXCVIII.

REVIEW.

1. Repeat the principles of the Roman system of notation.

2. Name the periods given in the Arabic numeration table.

SUBTRACTION.

Oral.

INDUCTIVE STEPS.

1. How many are:
   1. 4 apples less 2 apples?
   2. 5 dollars less 3 dollars?
   3. 6 birds less 3 birds?
   4. 7 cents less 4 cents?
   5. 10 dollars less 5 dollars?
2. What is the difference:
   1. Between 9 years and 6 years?
   2. Between 10 days and 8 days?
   3. Between 11 dollars and 7 dollars?
   4. Between 11 dollars and 7 yards?

How must the question be changed that the difference may be found?

Between what kind of numbers can we find a difference?

3. The difference between 11 and 7 is four. If you add the difference 4 to the less number, what number do you obtain?

4. What is the difference between 12 days and 7 days?

How can you prove that 5 days is the correct result?

Is not finding the difference between 12 and 7 the same as subtracting 7 from 12?

DEFINITIONS.

1. **Subtraction** is the process of finding the difference between two numbers. In subtraction one number is said to be taken from another.

2. The **Minuend** is the number from which another number is to be subtracted.

3. The **Subtrahend** is the number to be subtracted.

4. The **Remainder** or **Difference** is the result obtained.

5. The **Sign of Subtraction** is a short horizontal line (—) called **minus** (less); it is placed between the two numbers to be subtracted, and shows that the one after it is to be subtracted from the one before it.

11 — 7 = 4, is read, "11 minus 7 equals 4."

11 — 7 = 4, being an expression of equality, is called what?

PRINCIPLES.

1. The minuend and subtrahend must be like numbers.

2. The sum of the subtrahend and remainder equals the minuend.

DRILL.

Complete, at sight, the following equations:

1.	4 — 1 =	5 — 2 =	7 — 2 =	10 — 7 =
2.	4 — 3 =	5 — 4 =	9 — 6 =	9 — 4 =
3.	4 — 2 =	5 — 3 =	9 — 3 =	8 — 5 =
4.	4 — 4 =	5 — 5 =	9 — 9 =	8 — 8 =

From	10	9	6	11	12	14	16	15	10	9
Subtract	6	7	3	10	5	7	6	8	4	7

From	20	18	13	13	14	15	11	10	8	13
Subtract	10	9	8	9	6	7	9	8	7	5

From	12	19	18	19	15	13	16	18	20	14
Subtract	6	10	9	9	8	7	4	6	2	6
From	15	16	13	11	12	12	13	11	15	15
Subtract	5	8	5	7	8	7	6	8	6	12
From	16	16	17	17	18	18	19	19	20	20
Subtract	6	9	8	5	5	10	15	7	3	9

ORAL PROBLEMS.

1. A boy solved 12 examples on Monday and 15 on Tuesday. How many more did he solve on Tuesday than on Monday?

2. Mr. A. had 20 sheep, but sold 10. How many were left?

3. Henry had 50 apples; he gave 30 of them to Charles. How many had he left?

4. A merchant had 19 metres of cloth and sold 8 metres. How many metres had he left?

5. John had 28 cents, but gave 8 to James. How many remained?

6. Jenkins is 80 years old and his son is 50. How much older is Jenkins than his son?

7. Seventeen criminals escaped from a jail, but 8 of them were caught. How many secured freedom?

8. A boy gathered 13 quarts of berries and sold 4 quarts. How many did he keep?

9. A girl who had 20 cents bought nuts for 8 cents. How much money remained?

10. If you should buy a cow for $20 and sell her for $30, how much would you gain?

11. How much would I lose if I bought a wagon for $45 and sold it for $25?

12. If $1.00 is the minuend and 75 cents the subtrahend, what is the remainder?

13. If 75 cents is the subtrahend and 25 cents the remainder, what is the minuend?

14. What is the second principle of subtraction?

15. I am now 58 years old. How old was I 15 years ago?

16. A librarian loaned 21 books; all were novels but 8. How many were novels?

17. 58 men joined the army; only 23 returned home. How many were unable to return?

18. The United States flag displays 45 stars; originally but 13. What has been the increase?

19. How many are:

1. 48 — 17?	8. 41 — 20?	15. 49 — 13?
2. 39 — 18?	9. 36 — 16?	16. 44 — 33?
3. 28 — 7?	10. 39 — 9?	17. 39 — 19?
4. 59 — 19?	11. 48 — 11?	18. 48 — 18?
5. 49 — 27?	12. 44 — 14?	19. 46 — 26?
6. 42 — 12?	13. 38 — 14?	20. 39 — 9?
7. 49 — 19?	14. 37 — 30?	21. 49 — 11?

20. Count by 2's from 50 to 2; from 100 to 50.

21. Count by 5's from 100 to 60; from 60 to 20.

22. Count by 4's from 100 to 40; from 40 to 0.

23. Count by 6's from 90 to 60; from 60 to 12.

All the figures of the minuend may be of greater value than the corresponding figures of the subtrahend; and one or more figures of the minuend may have *less value.*

Figures of the Minuend All of Greater Value.

WRITTEN EXERCISES.

1. From 978 take 534.

Process.

Minuend 978
Subtrahend 534
Remainder 444

Explanation.

Since the unit of the numbers is simply *one* without name, the numbers are *like* and their difference may be found.

We write units of the same order in the same column, and, beginning on the right, say 8 units — 4 units = 4 units; 7 tens — 3 tens = 4 tens; 9 hundreds — 5 hundreds = 4 hundreds. Hence, the remainder is 444.

Proof.

444 + 534 = 978.

Repeat the two principles of subtraction.
What is the unit of 978 horses? Of 534 oxen?
Are the numbers, then, like or unlike?
Can their difference be found?

2. Subtract and prove the following:

(1)	(2)	(3)	(4)	(5)	(6)	(7)
536	972	654	890	904	529	975
325	420	313	460	402	328	342

(8)	(9)	(10)	(11)	(12)	(13)	(14)
968	430	900	675	999	951	938
627	220	500	324	467	511	627

(15)	(16)	(17)	(18)	(19)	(20)
3976	9803	6975	9795	8396	9457
2633	5603	3762	6254	5163	2346

(21)	(22)	(23)	(24)	(25)	(26)
$64.37	$59.40	$49.95	$59.99	$98.68	$99.87
42.26	35.20	33.82	46.54	62.54	46.52

(27)	(28)	(29)	(30)	(31)
$93.27	$95.125	$98.96	$59.676	$635.89
32.15	24.125	53.06	49.250	25.72

WRITTEN PROBLEMS.

1. A farm is worth $896, and a house is worth $194. How much more is the farm worth than the house?

NOTE.—Indicate all processes, thus: $896 — $194 = dollars required.

2. A boy had $1.50; he bought a book for 50 cents. How much money had he left?

3. A woman went to market with a 5-dollar bill; on her return she had a 2-dollar bill and 50 cents change. How much had she spent?

4. A man bought 9893 bricks, but used only 6473. How many had he left?

5. Gold was discovered in California in 1848. How long ago?

6. Lucifer matches were first made by machinery in 1850. How long ago was that?

7. I bought a farm for $4796.30 and sold it for $4897.50. How much did I gain?

8. A clerk earned in a year $998.45 and spent $746.30. How much did he save?

9. A brigade going into battle numbered 6975 men; after the battle only 3844 responded at roll-call. How many had the battle removed?

10. Mr. Hewitt bought a city lot for $2986 and afterward sold it for $3986. How much did he gain?

11. A ship that cost $7680 was sold for $5650. How great was the loss?

12. A.'s income for a year is $9874. How much does he save if his expenses are $4370?

13. Suppose I had lent a man 1565 dollars and he died owing me 450 dollars. How much had he paid me?

14. A person sold a farm for 15,896 dollars, which had cost him 12,264 dollars. How much did he gain?

15. Washington died in 1799, at the age of 67. In what year was he born?

16. The World's Columbian Exposition at Chicago was held in 1893. How many years after the discovery of America by Columbus?

17. Lanterns were invented by King Alfred in 890. How long ago?

18. A grocer received for sugar $308.40; gained $106.20. Find the cost.

19. How much nearer is 1492 to 1792 than to 1898?

ORAL EXERCISES.

1. Subtract by:

1. 5's from 25, saying 25, 20, 15, etc.
2. 10's from 50 to 0. From 100 to 0.
3. 4's from 20 to 0. From 30 to 2.
4. 3's from 20 to 2. From 30 to 0.
5. 2's from 20 to 0. From 30 to 0.
6. 6's from 20 to 2. From 30 to 0.
7. 7's from 20 to 6. From 30 to 2.
8. 8's from 20 to 4. From 30 to 6.

9. 9's from 20 to 2. From 30 to 3.
10. 6's from 52 to 4. From 53 to 5.

2. How many are:

1. 25 — 5? 35 — 5? 45 — 5? 55 — 5?
2. 52 — 4? 62 — 4? 72 — 4? 82 — 4?
3. 63 — 6? 73 — 6? 83 — 6? 93 — 6?
4. 74 — 8? 84 — 8? 94 — 8? 24 — 8?
5. 85 — 9? 95 — 9? 75 — 9? 65 — 9?
6. 31 — 2? 41 — 2? 51 — 2? 61 — 2?
7. 22 — 3? 32 — 3? 42 — 4? 52 — 5?
8. 44 — 7? 54 — 7? 64 — 7? 74 — 7?
9. 39 — 9? 99 — 9? 109 — 9? 110 — 9?
10. 33 — 9? 43 — 8? 53 — 7? 53 — 8?

Figures of Less Value in the Minuend.

WRITTEN EXERCISES.

1. From 110 subtract 9.

Process.

$$\begin{array}{r} 110 \\ 9 \\ \hline 101 \end{array}$$

Explanation.

110 — 9 = 101, because 101 + 9 = 110; but we may reason thus, saying: "We cannot subtract 9 units from 0 units, but the 1 ten = 10 units, and 10 units — 9 units = 1 unit, which we write in the remainder. Having used the 1 ten, we write 0 in the remainder, and also the 1 hundred which we did not use. Hence 110 — 9 = 101."

The following is the extended process and explanation:

110 = 1 hundred + 1 ten + 0 units.
9 = 9 units.

By reducing the 1 ten to units, the above becomes:

110 = 1 hundred + 0 tens + 10 units.
9 = 9 units. Subtracting, we have 1 hundred + 0 tens + 1 unit = 101.

2. From 734 subtract 389.

Process.		Explanation.
Minuend	734	We have written the less number under the greater, units under units, and tens under tens, etc. 3 and 4 of the minuend are of less value than 8 and 9 of the subtrahend; 9 cannot be taken from 4. The 4 must be added to in some way. One of the
Subtrahend	389	
Remainder	345	

3 tens = 10 units; 10 units + 4 units = 14 units; 14 units − 9 units = 5 units. Instead of 3 tens we have now but 2 tens; 8 cannot be taken from 2. One of the 7 hundreds = 10 tens; 10 tens + 2 tens = 12 tens; 12 tens − 8 tens = 4 tens. Instead of 7 hundreds we have now but 6 hundreds; 6 hundreds − 3 hundreds = 3 hundreds. Hence, the remainder is 345.

Proof.

345 + 389 = 734.

The extended process and explanation are as follows:

734 = 7 hundreds + 3 tens + 4 units.
389 = 3 hundreds + 8 tens + 9 units.

The tens' and the units' figures of the minuend being of less value than the tens' and units' figures of the subtrahend, it becomes necessary to reduce one of the hundreds to tens, and one of the tens to units. By doing this we have

734 = 6 hundreds + 12 tens + 14 units.
389 = 3 hundreds + 8 tens + 9 units.

Subtracting, we have

3 hundreds + 4 tens + 5 units = 345.

$$\begin{array}{r} {\scriptstyle 6\ 12\ 14} \\ 7\,3\,4 \\ 3\,8\,9 \\ \hline 3\,4\,5 \end{array}$$

Instead of extending the work as above, we may make the reductions and indicate the changes by placing small figures above the figures of the minuend, as in the margin.

NOTE.—As soon as pupils understand how to make the reductions, these helps should be omitted.

TO THE TEACHER.—Herein lies the chief difficulty of subtraction. Pupils should be drilled in both process and explanation until they thoroughly understand the work and can perform it with facility. Do not pass hurriedly over the fundamental operations; keep them before the pupils long enough to make an impression. Good work here means sure and rapid progress hereafter. Lay a good foundation.

On what principle does the proof depend?

On what principle does the above process depend?

Brief directions are:

1. Write the subtrahend under the minuend, units under units, tens under tens, etc.

2. Begin at the right to subtract.

3. Take each figure of the subtrahend from the figure above it in the minuend.

4. Add 10 to a figure of less value in the minuend and take one from the next figure on the left in the minuend.

5. Prove by adding the remainder to the subtrahend.

3. Subtract and prove the following:

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
546	792	695	725	938	592	493	827
377	580	304	396	674	205	378	795

(9.)	(10.)	(11.)	(12.)	(13.)	(14.)	(15.)	(16.)
313	704	630	357	529	912	845	756
247	195	548	249	483	345	678	680

(17.)	(18.)	(19.)	(20.)	(21.)	(22.)	(23.)
$44.35	$35.29	$42.35	$49.42	$34.92	$79.25	$94.37
23.96	27.37	25.87	28.37	25.67	43.87	58.46

(24.)	(25.)	(26.)	(27.)	(28.)	(29.)
$104.21	$549.37	$345.93	$345.30	$1786.08	$3545.37
75.80	99.89	76.04	187.23	1097.19	966.38

4. From 3000 subtract 958.

Process.	Explanation.
3000 958 2042	We cannot subtract 8 from 0. We therefore take 1 ten from the 300 tens, leaving 299 tens. 1 ten = 10 units; 10 units − 8 units = 2 units. Instead of 300 tens we have now 299 tens; 95 tens from 299 tens leaves 204 tens. Hence, the remainder is 2042.

Proof.

$2042 + 958 = 3000.$

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
3507	4709	4000	5000	70000	80000	90000	70000
2564	2678	2565	3794	34567	78910	12345	68904

(9.)	(10.)	(11.)	(12.)	(13.)	(14.)
100000	1000000	1000	$1000.00	$234.56	$93000000
99999	1000	407	874.23	87.678	240000

5. Subtract:

1. 404 from 795.
2. 390 from 807.
3. 567 from 954.
4. 954 from 1005.
5. 8376 from 10367.
6. 4638 from 9524.
7. 4076 from 5679.
8. 4005 from 5967.
9. 3786 from 6004.
10. 4976 from 69005.
11. 3679 from 38050.
12. 40576 from 580676.
13. 30794 from 490684.
14. 430576 from 5600750.
15. 767374 from 903727.
16. 7503706 from 8732314.
17. 758386 from 890573.
18. 3885238 from 8630572.
19. 8735918 from 9043057.
20. 7953240 from 9932563.

WRITTEN PROBLEMS.

1. Find the difference between twenty thousand two and twelve thousand eight hundred.

2. Find the value of one million seven hundred less forty-nine thousand nine.

3. In 1892 a city had 58,376 inhabitants. This number was greater than that of the previous year by 5795. What was the number of the previous year?

4. A man bought a house for $7250 and sold it at a loss of $788. How much did he sell it for?

5. A man has $1980. How much does he lack of having enough to buy a farm worth $13,000?

6. A debtor owing $10,847, paid $4090. How much did he still owe?

7. The seeds in a bushel of rye number 888,390; in a bushel of wheat, 556,290. Find the excess of rye seeds.

8. How much must be added to 109 to make 1,000,000?

9. In 1895 the production of wheat in Europe was 1,443,000,000 bushels; in the United States, 467,000,000 bushels. How many more bushels were produced in Europe than in the United States?

10. In 1895 Iowa produced 298,503,000 bushels of corn, exceeding in this respect any other State; Montana, in the same year, 33,000 bushels. Find the difference.

11. In 1895 California produced 40,098,000 bushels of wheat; Kansas, 29,919,000 bushels; Pennsylvania came third, with 20,456,000 bushels. How far did California lead each of the two other States?

12. In Hamilton County, Ohio, the internal revenue tax on tobacco in 1894 was $1,375.32; in 1895, $1,170.36. Find the decrease.

13. In Cincinnati the mean temperature of summer is 76 degrees; the mean temperature of winter is 29.2 degrees. Find the difference.

14. Sound travels through the air at the rate of 1090 feet per second when the temperature is 32 degrees, and 1 foot faster for every degree the temperature rises. How fast per second does sound travel when the temperature is 70 degrees?

ADDITION AND SUBTRACTION.

ORAL EXERCISES.

How many are:

1. $4 + 3 - 5 + 4 + 6 + 0 - 3 - 5$? Say 4, 7, 2, 6, 12, 9, result 4.
2. $7 - 1 + 4 - 5 + 7 - 3 - 8 + 9 - 4 + 5 + 8$?
3. $9 + 3 - 5 + 7 - 4 + 3 - 6 + 8 - 4 + 7 - 3$?
4. $6 + 5 - 3 - 4 + 7 - 3 + 4 - 5 + 7 - 6 + 8$?
5. $4 + 7 - 3 + 8 - 6 + 7 - 9 + 6 - 5 + 0 - 8$?
6. $9 + 6 - 5 + 7 - 4 - 3 + 7 + 5 - 5 - 4 + 5$?
7. $8 + 3 + 4 - 7 + 5 - 6 + 3 - 8 + 4 + 7 + 5$?
8. $9 + 3 - 7 - 4 + 7 - 3 + 5 - 6 + 7 - 7 + 3$?
9. $9 + 4 - 6 + 7 + 8 - 4 + 7 - 4 - 7 + 3 + 6$?
10. $9 + 3 - 7 + 4 - 3 - 2 + 9 - 5 - 4 + 7 + 3$?
11. $9 + 3 - 7 + 4 - 3 - 2 + 9 - 5 - 4 + 8 + 3$?
12. $10 - 4 + 8 - 5 - 6 + 4 + 5 - 3 - 6 + 5 + 4 + 6$?
13. $11 - 1 - 2 - 3 - 4 - 1 + 1 + 2 + 3 + 4 + 5 + 6$?
14. $12 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + 9 - 0 + 1 - 2 + 3$?
15. $13 - 4 + 5 - 6 + 7 - 8 + 9 - 1 + 2 - 3 + 4 - 5 + 6$?
16. $14 - 5 + 6 - 7 + 8 - 9 + 1 - 2 + 3 - 4 + 5 - 6 + 7$?
17. $15 - 6 + 7 - 8 + 9 - 1 + 2 - 3 + 4 - 5 + 6 - 7 + 8$?
18. $16 - 7 + 8 - 9 + 1 - 2 + 4 - 5 + 6 - 7 + 8 - 9$?
19. $17 - 8 + 9 - 8 + 7 - 6 + 1 - 2 + 3 - 4 + 5 - 6 + 7$?

WRITTEN PROBLEMS.

1. Sarah had 5 dollars and earned 10 more; she then spent 8 dollars. How much money had she left?

2. Margaret wrote 12 letters in the forenoon and 14 in the afternoon. If she mailed 16 of them, how many had she still on hand?

3. William owned 16 rabbits. He sold 4 to Thomas, 6 to Henry, and bought 3 from James. How many rabbits had he then?

4. On leaving Eighth Street a trolley car had 6 passengers; at Fourteenth Street 2 left it and 6 entered it; at Twenty-second Street 1 left and 4 entered; at Thirty-seventh Street 4 more entered; at Forty-second Street 2 left. How many now remained?

5. A merchant had a piece of cloth containing 54 yards. He sold 12 yards to one man, 15 to another, and 10 to another. How many yards remained in the piece?

6. A gentleman bought a watch for $70 and a chain for $15. He sold the two for $97. How much did he gain?

7. A farmer had 70 sheep in one field and 65 in another. He sold 35 from each field. How many remained?

8. Virginia bought a shawl for $16 and a pair of gloves for $2. She gave in payment two $10 bills. How much change was due her?

9. A man having received $56 for his services, paid $25 for a coat, $6 for a barrel of flour, and $5.50 for a ton of coal. How much money had he left?

10. The difference of the ages of two persons was 49

years; the younger person was born in 1850. In what year was the older born?

11. James has 97 cents; he pays 37 cents for a whistle and 60 cents for a knife. How much has he left?

12. Two men start from the same place and travel in the same direction. When one has travelled 65 miles and the other 47 miles, how far will they be apart?

13. John had 31 marbles; his father gave him 9 more; he sold 7, found 4, lost 8, and from his brother received in trade 4 for 5. How many did he then have?

14. A farmer paid $35 for a cow, $15 for sheep, $20 for pigs, and exchanged them all for a hundred-and-fifty-dollar horse. How much did he have to pay in cash?

15. A man went into a clothing-store and bought a vest for $6.00, a coat for $20, and pantaloons for $11. He handed the clerk 4 ten-dollar bills. How much did he receive in change?

WRITTEN REVIEW.

1. What is an equation?
2. Finish the following equations:
   1. $8975 + 4308 - 5904 + 3275 =$
   2. $4927 - 3049 + 5574 - 5267 =$
   3. $9240 + 3796 - 5432 - 5237 =$
   4. $24570 + 27957 - 4907 + 3275 =$
   5. $37920 - 5970 - 24357 + 3795 =$
   6. $46249 - 10987 - 9854 - 24523 =$
   7. $4062 + 12356 + 15000 - 975 =$
   8. $23462 + 9030 + 34000 - 7640 =$
   9. $19876 - 6032 - 12000 + 673 =$
   10. $87642 + 798764 - 379862 - 4001 - 14760 =$

3. Find the difference between twenty thousand two and twelve thousand eight hundred.

4. A man has $10,000. How much must he borrow that he may be able to pay $24,750 for an estate?

5. Three men bought a building for $47,956. If the first paid $21,706, and the second $9575, how much remained for the third to pay?

6. If I deposit in one bank $1095.54, in another $987.95, and in a third $709.28, how much must I borrow to enable me to build a house worth $5486?

7. The sum of three numbers equals 98,765; two of the numbers are 28,907 and 36,794; what is the third number?

8. A man sold a firkin of butter for $20, a cheese for $10, and a quantity of fruit for $12.50. He received in payment 5 barrels of flour valued at $25.75. How much is still owing to him?

9. A farmer, having 625 bushels of grain, sold to A. 97 bushels, to B. 127, to C. 197, and gave 110 bushels to the poor. How many bushels had he remaining?

10. A man deposited $10,000 in a bank. He drew out at one time $47.00; at another, $24.80; and at another, $1474. How much had he remaining in the bank?

11. A boy bought a sled for $2.60, and gave $1.00 for having it repaired. He sold it for $4.00, and lost $2.50 of the money. To what extent was he then out of pocket?

12. I bought 28 yards of cloth for $26.54, 10 yards of calico for $2.72, 25 pounds of sugar for $1.375, 4 pounds of tea for $1.95. I paid $9.85. How much do I still owe?

13. A. had 450 sheep, B. had 175 more than A., and C. had as many as A. and B. together minus 114. How many sheep had C.?

14. Mr. Swift has in cash $419.14, but he owes Mr. Brick $47.55, Mr. Quick $274.30, and Mr. Lick $97.29. After paying these gentlemen, how much will he have left?

15. A. paid $9000 for his farm, $8476 for a new house, and $873 for a barn, and then sold them for $29,600. What was his gain?

16. In 1890 the exports of the United States amounted to $857,828,684, the imports to $789,310,409. How much does the sum of these two amounts exceed their difference?

17. A farmer had 738 sheep. He sold 327 to A. and 234 to B. Disease carried off 90. How many remained?

18. I paid $2240 for some land. I sold coal for $116, a horse for $225, a wagon with a horse attached for $277, and collected a bill of $565.25 long due. How much more did I pay out than I received?

19. If February has 28 days, and April, June, September and November have 120 days, the remaining seven months supply how many days to make up 365 days?

20. If June (30 days), July (31 days), and August (31 days) are the summer months, and December (31 days), January (31 days), and February are the winter months, which is the longer, summer or winter, and how much?

21. I start out with $205.50 in one pocket and $43.25 in the other. I pay the grocer $63.69, the butcher $32.08

the shoemaker $10.70, the landlord $37.50, the tailor $17.50. How much have I left?

22. Margaret has 74 cents, Mary has 135 cents. Mary gave Margaret 28 cents. Which has then the greater sum, and greater by how much?

23. If the minuend is $4937 and the remainder is $1593, what is the subtrahend? The subtrahend being $3825 and the remainder $337.84, what is the minuend?

24. From what number must I subtract 10 to leave 14? 16 to leave 18? 2.586 to leave 4.098?

25. What sum must be subtracted from $1.00 to leave $.50? To leave $.625? To leave $.375? To leave $.875?

26. What number increased by 63,915 makes a million?

27. If in July 220,860 carriages visited Fairmount Park, 5575 equestrians, and 1,443,173 pedestrians, find the excess of pedestrians over the other two classes.

28. A lady bought a bonnet for $12, a pair of shoes for $4, and a fan for $1. She gave the salesman a $20 bill. What change did she receive?

29. Find the value of MCLX. + LXXXVIII. — DXL. + IX. + IV. — XC.

30. Find the value of MCMXCVIII. — XCVIII. — DCCC. — M.

31. A man had in bank $15,000, deposited $3875, drew out $8725, and then put in enough to make his deposit $20,000. How much did he last put in?

32. $15,000 is divided among three children, the second of whom receives $2240 more than the first, and the third of whom receives the remainder. If the first receives $5000, how much does each of the others receive?

MULTIPLICATION.

INDUCTIVE STEPS.

1. How many are:

1. $1 + 1$? $4 + 4$?
2. $2 + 2$? $5 + 5$?
3. $3 + 3$? $6 + 6$?

2. Hence, we may say:

1. Two times 1 = 2. Two times 4 = 8.
2. Two times 2 = 4. Two times 5 = 10.
3. Two times 3 = 6. Two times 6 = 12.

3. Instead of writing:

1. $1 + 1 + 1 = 3$. $4 + 4 + 4 = 12$.
2. $2 + 2 + 2 = 6$. $5 + 5 + 5 = 15$.
3. $3 + 3 + 3 = 9$. $6 + 6 + 6 = 18$, etc.

We can write more briefly:

1. Three times 1 = 3. Three times 4 = 12.
2. Three times 2 = 6. Three times 5 = 15.
3. Three times 3 = 9. Three times 6 = 18, etc.

$2 \times 1 = 2$ is read "two times one equal two"; the word *times* being expressed by the sign.

The process of thus shortening addition is called **Multiplication.**

4. Proceeding in like manner with 4, 5, 6, 7, 8, 9, 10, 11, 12, and 13, we construct the following table:

Multiplication Table.

2 × 1 = 2	3 × 1 = 3	4 × 1 = 4	5 × 1 = 5
2 × 2 = 4	3 × 2 = 6	4 × 2 = 8	5 × 2 = 10
2 × 3 = 6	3 × 3 = 9	4 × 3 = 12	5 × 3 = 15
2 × 4 = 8	3 × 4 = 12	4 × 4 = 16	5 × 4 = 20
2 × 5 = 10	3 × 5 = 15	4 × 5 = 20	5 × 5 = 25
2 × 6 = 12	3 × 6 = 18	4 × 6 = 24	5 × 6 = 30
2 × 7 = 14	3 × 7 = 21	4 × 7 = 28	5 × 7 = 35
2 × 8 = 16	3 × 8 = 24	4 × 8 = 32	5 × 8 = 40
2 × 9 = 18	3 × 9 = 27	4 × 9 = 36	5 × 9 = 45
2 × 10 = 20	3 × 10 = 30	4 × 10 = 40	5 × 10 = 50
2 × 11 = 22	3 × 11 = 33	4 × 11 = 44	5 × 11 = 55
2 × 12 = 24	3 × 12 = 36	4 × 12 = 48	5 × 12 = 60
6 × 1 = 6	7 × 1 = 7	8 × 1 = 8	9 × 1 = 9
6 × 2 = 12	7 × 2 = 14	8 × 2 = 16	9 × 2 = 18
6 × 3 = 18	7 × 3 = 21	8 × 3 = 24	9 × 3 = 27
6 × 4 = 24	7 × 4 = 28	8 × 4 = 32	9 × 4 = 36
6 × 5 = 30	7 × 5 = 35	8 × 5 = 40	9 × 5 = 45
6 × 6 = 36	7 × 6 = 42	8 × 6 = 48	9 × 6 = 54
6 × 7 = 42	7 × 7 = 49	8 × 7 = 56	9 × 7 = 63
6 × 8 = 48	7 × 8 = 56	8 × 8 = 64	9 × 8 = 72
6 × 9 = 54	7 × 9 = 63	8 × 9 = 72	9 × 9 = 81
6 × 10 = 60	7 × 10 = 70	8 × 10 = 80	9 × 10 = 90
6 × 11 = 66	7 × 11 = 77	8 × 11 = 88	9 × 11 = 99
6 × 12 = 72	7 × 12 = 84	8 × 12 = 96	9 × 12 = 108
10 × 1 = 10	11 × 1 = 11	12 × 1 = 12	13 × 1 = 13
10 × 2 = 20	11 × 2 = 22	12 × 2 = 24	13 × 2 = 26
10 × 3 = 30	11 × 3 = 33	12 × 3 = 36	13 × 3 = 39
10 × 4 = 40	11 × 4 = 44	12 × 4 = 48	13 × 4 = 52
10 × 5 = 50	11 × 5 = 55	12 × 5 = 60	13 × 5 = 65
10 × 6 = 60	11 × 6 = 66	12 × 6 = 72	13 × 6 = 78
10 × 7 = 70	11 × 7 = 77	12 × 7 = 84	13 × 7 = 91
10 × 8 = 80	11 × 8 = 88	12 × 8 = 96	13 × 8 = 104
10 × 9 = 90	11 × 9 = 99	12 × 9 = 108	13 × 9 = 117
10 × 10 = 100	11 × 10 = 110	12 × 10 = 120	13 × 10 = 130
10 × 11 = 110	11 × 11 = 121	12 × 11 = 132	13 × 11 = 143
10 × 12 = 120	11 × 12 = 132	12 × 12 = 144	13 × 12 = 156

5. In the column beginning with $4 \times 1 = 4$, show how all the results, 4, 8, 12, 16, etc., were obtained.

6. The multiplication table is an invention of the greatest value. If you do not yet know this table, learn it perfectly as soon as possible.

DEFINITIONS.

1. **Multiplication** is a short process of adding when the numbers to be added are *all equal;* or, it is the process of finding the result of repeating one number as many times as there are units in another.

2. The **Multiplicand** is the number to be repeated or multiplied.

3. The **Multiplier** is the number by which we multiply.

4. The **Product** is the result of multiplying.

5. The Multiplicand and Multiplier are called **Factors** of the product.

6. We may write 5 times 12 equal 60, or we may write 5 times 12 men equal 60 men, or five times \$12 equal \$60. In the first case, 12 and 60, having no application to men, or money, or other things, are called **Abstract Numbers**; in the second case, being applied to men, and in the third case to dollars, 12 and 60 are called **Denominate Numbers.**

The multiplier 5 is *abstract* in all the cases; showing no more than the number of times 12 is to be taken.

7. Since $5 \times 12 = 60$, and $12 \times 5 = 60$, we see that the product is the same whether 5 or 12 is used as the multiplier.

PRINCIPLES.

1. The multiplier must be used as an abstract number.

2. The multiplicand and the product are like numbers.

3. Either factor, when abstract, may be used as multiplier or multiplicand.

ORAL EXERCISES.

1. Multiply	9	6	7	9	7	6	5	8	9	6	9	4	8
By	5	4	8	6	0	9	7	9	4	8	6	9	6
2. Multiply	7	8	6	5	8	7	6	5	8	5	6	8	9
By	5	4	7	9	6	7	4	8	4	8	7	5	6
3. Multiply	0	2	3	4	5	6	7	8	9	1	2	3	4
By	9	8	7	6	5	7	8	8	9	8	7	8	9

4. Multiply	10	11	12	13	9	10	8	11	7	12	6	13
By	5	4	6	7	8	9	10	11	12	7	6	5

Is $8 \times 9 = 9 \times 8$ a true equation?

By what name are 8 and 9 called in their relation to the product?

ORAL PROBLEMS.

1. A dime equals 10 cents. 8 dimes equal how many cents?

2. If 6 hours make a school day, how many hours in 5 school days?

3. How much will 2 pencils cost at 5 cents each?

4. There are 7 days in a week. How many days in 6 weeks?

5. How many are 9 times 5 pears? 5 times 9 pears?

6. How much can a man earn in 11 days at $5 a day?

7. How much will 3 cows cost at $13 each?

8. What is the cost of 2 oranges at 5 cents each and of 4 bananas at 3 cents each?

9. How many days in 12 weeks?

10. How much must I pay for 9 sheep at $7 a head?

11. If 5 men can do a piece of work in 3 days, how long will it take 1 man?

12. If 12 men can do a piece of work in 10 days, how many men will be required to do it in 1 day?

13. How many are $6 \times 4 + 7$? $8 \times 5 + 30$?

14. How many are $5 \times 6 + 10 - 25$?

15. If a boy earns $7 a month, how much can he earn in 9 months?

16. At 2 cents a foot, how much will 13 feet of board cost?

17. Repeat the first, second, third, and fourth columns of the multiplication table.

18. Repeat the fifth, sixth, seventh, and eighth columns of the table.

19. Repeat the ninth, tenth, eleventh, and twelfth columns of the table.

20. Declare promptly the product of:

1. 2×11.	5. 5×8.	9. 8×4.	13. 10×9.
2. 3×10.	6. 6×7.	10. 9×3.	14. 11×11.
3. 4×9.	7. 7×5.	11. 7×7.	15. 12×6.
4. 4×7.	8. 5×9.	12. 9×9.	16. 2×12.

The Multiplier a Single Figure.

WRITTEN EXERCISES.

1. How many are 5 times 1728?

Process.			Explanation.
Multiplicand,	1728	} Factors.	1. 5 times 8 units = 40 units = 4 tens + *0 units.*
Multiplier,	5		2. 5 times 2 tens = 10 tens; 10 tens + 4 tens = 14 tens = 1 hundred + *4 tens.*
Product,	8640		

3. 5 times 7 hundreds = 35 hundreds; 35 hundreds + 1 hundred = 36 hundreds = 3 thousands + *6 hundreds.*

4. 5 times 1 thousand = 5 thousands; 5 thousands + 3 thousands = *8 thousands.*

Hence, the product is 8640.

Proof.

$1728 + 1728 + 1728 + 1728 + 1728 = 8640.$

2. Find the product of the following factors:

1. 347×5.	12. 6038×7.	23. 42598×8.
2. 357×6.	13. 7984×9.	24. 30487×7.
3. 530×4.	14. 6346×8.	25. 123456×2.
4. 937×4.	15. 5396×9.	26. 789012×3.
5. 348×5.	16. 3140×7.	27. 345678×4.
6. 905×3.	17. 53645×9.	28. 901234×5.
7. 570×6.	18. 30724×6.	29. 567890×6.
8. 943×5.	19. 42304×9.	30. 123456×7.
9. 3072×4.	20. 53728×7.	31. 890123×8.
10. 5937×7.	21. 43954×9.	32. 456789×9.
11. 8945×6.	22. 31708×8.	33. 876543×5.

WRITTEN PROBLEMS.

1. What will 9 pairs of shoes cost at $3.50 a pair?

2. At $25 each, what will 8 cows cost?

3. At $.50 each, what will 5 books cost?

4. In one square foot there are 144 square inches. How many square inches in 6 square feet?

5. There are 365 days in a year. How many days in 8 years?

6. If sound travels 1120 feet in a second, how far does it travel in 9 seconds?

7. In one bushel there are 2150.42 cubic inches. How many cubic inches in 7 bushels?

8. There are 5280 feet in a mile. How many feet in 6 miles?

9. What is the value of:

1. $8 \times 5 + 6 + 10$?
2. $5 \times 9 - 5 \times 6$?
3. $60 - 6 \times 1 + 8$?
4. $9 \times 5 - 7 \times 1$?
5. $5 \times 9 + 4 + 6$?
6. $0 \times 7 + 8 + 8$?

Suggestion: Multiply before adding or subtracting.

7. $13 \times 6 + 4 + 8 \times 2$?
8. $10 \times 8 + 10 - 7$?
9. $39 - 30 + 5 \times 9$?
10. $73 + 8 - 9 \times 9$?

10. Find

The sum of:	The product of:	The difference of:
57837	1. 729127 × 6.	1. 7000305 and 663650.
786584	2. 832561 × 7.	2. 9043057 and 873598.
828763	3. 563656 × 8.	3. 9540371 and 7936597.
683784	4. 624526 × 9.	4. 7987396 and 6798309.
148528	5. 536465 × 7.	5. 7799458 and 5988799.
691789	6. 563464 × 8.	6. 8735009 and 7295394.
979897	7. 399978 × 6.	7. 6937254 and 2786927.

The Multiplier with Ciphers Annexed.

INDUCTIVE STEPS.

1. What is the product of the following factors?

1. 5 and 10? 6 and 10? 7 and 10?

2. 10 and 5? 10 and 6? 10 and 7?

The significant figure in each product has what annexed to it?

Annexing a cipher, then, multiplies by what?

2. 8 times 100 = what? 100 times 8 = what?

The significant figure 8 has how many ciphers annexed in the product?

3. 9×1000 = what? 1000×9 = what?

The significant figure 9 in the product has how many ciphers annexed?

PRINCIPLES.

1. Annexing 1 cipher multiplies by 10.

2. Annexing 2 ciphers multiplies by 100.

3. Annexing 3 ciphers multiplies by 1000, and so on.

WRITTEN EXERCISES.

1. Multiply 144 by 10.

Process.

Multiplicand, 144 } Factors.
Multiplier, 10 }
Product, 1440

Explanation.

In accordance with Principle 1, to multiply by ten, we annex one cipher to the multiplicand, and obtain for product 1440.

Or, we may say: "144 times 1 ten = 144 tens; but 144 tens = 1440 units. Hence, the product is 1440."

2. Multiply 365 by 1000.

Process.

365
1000 } Factors.
365000

Explanation.

In accordance with Principle 3, to multiply by 1000 we annex three ciphers to the multiplicand, and obtain for product 365,000.

Or, we may say: "365 times 1 thousand = 365 thousands; but 365 thousands = 365,000 units. Hence, the product is 365,000."

3. Multiply 1728 by 3000.

Process.

1728
3000
5184000

Explanation.

3000 = 3 times 1000; 1728 multiplied by 1000 = 1,728,000; 1728 multiplied by 3000 = 3 times 1,728,000, or 5,184,000.

Or, we may say: "1728 times 3 thousands = 5184 thousands; but 5184 thousands = 5,184,000 units. Hence, the product is 5,184,000."

RULE.

1. **Cut off the ciphers from the multiplier.**
2. **Multiply by the significant figure.**
3. **To the product annex the ciphers cut off.**

To apply the rule:

4. Multiply 1898 by 400.

Process.

1898
4|00
759200

Explanation.

Having cut off the two ciphers, we then multiply 1898 by 4 and obtain 7592. Annexing the two ciphers, we have 759,200.

5. Multiply:

(1)	(2)	(3)	(4)	(5)
579	480	269	3760	3405
100	1000	100	1000	100
(6)	**(7)**	**(8)**	**(9)**	**(10)**
4500	4000	5700	779	495
100	1000	1000	100	30

(11)	(12)	(13)	(14)	(15)
570	385	679	607	508
40	60	80	300	400

(16)	(17)	(18)	(19)	(20)
679	572	934	768	938
800	300	700	4000	5000

(21)	(22)	(23)	(24)	(25)
1136	922	1646	1336	2672
200	50	6000	70	9000

(26)	(27)	(28)	(29)	(30)
548	1136	592	1192	994
6000	7000	8000	9000	1000

The Multiplier Two or More Significant Figures.

1. Multiply 374 by 243.

Process.	Explanation.
375	We write the multiplier under the multiplicand, units
243	under units, etc.
1125	375 multiplied by 3 units = 1125 units. (Why?)
1500	375 multiplied by 4 tens = 1500 tens. (Why?)
750	375 multiplied by 2 hundreds = 750 hundreds. (Why?)
91125	Writing these three products in order as units, tens and hundreds, and adding, we have as total product 91,125.

2. Multiply 3703 by 408.

Process.	Explanation.
3703	8 times 3703 = 29,624 units.
408	0 times 3703 = 0 tens.
29624	4 times 3703 = 14,812 hundreds.
148120	Adding the partial products, we have 1,510,824.
1510824	

3. Multiply:

1. 370 by 43.
2. 596 by 35.
3. 704 by 45.
4. 836 by 54.
5. 937 by 63.
6. 938 by 46.
7. 937 by 35.
8. 925 by 66.
9. 530 by 74.
10. 739 by 59.
11. 832 by 68.
12. 740 by 86.
13. 7954 by 78.
14. 9836 by 85.
15. 9730 by 94.
16. 2397 by 99.
17. 3805 by 70.
18. 3496 by 68.
19. 3795 by 47.
20. 5368 by 72.
21. $46.96 by 46.
22. $27.30 by 78.
23. $35.29 by 40.
24. $35.46 by 54.
25. $34.10 by 76.
26. $59.42 by 79.
27. $29.68 by 80.
28. $38.96 by 43.
29. $29.875 by 50.
30. $68.57 by 65
31. 30796 by 245.
32. 49075 by 406.
33. 30986 by 570.
34. 39687 by 542.
35. 30425 by 341.
36. 37968 by 535.
37. 79846 by 270.
38. 57964 by 329.
39. 46854 by 213.
40. 67895 by 304.
41. 579656 by 2134.
42. 678956 by 3045.
43. 897362 by 4606.
44. 812357 by 4328.
45. 183586 by 2345.
46. 970890 by 7980.
47. 759671 by 9087.
48. 935802 by 9358.
49. 215090 by 7809.
50. 195833 by 9800.
51. 370184 by 10708.
52. 508199 by 25409.
53. 190199 by 89760.
54. 280755 by 20499.
55. 162788 by 72644.
56. 320988 by 23577.
57. 684579 by 323500.
58. 123456 by 789000.
59. 987643 by 210034.
60. 567890 by 123456.

WRITTEN PROBLEMS.

1. In one barrel of flour there are 196 pounds. How many pounds in 434 barrels?

2. A railway train travels an average of 235 miles per day. How far does it travel in 31 days?

3. There are 1760 yards in one mile. How many yards in 397 miles?

4. If a man travels 30 miles in one day, how many miles will he travel in 49 days?

5. A farmer raised 1448 bushels of wheat, which he sold at 76 cents per bushel. How much did he receive for it?

6. I sold my farm of 302 acres at $105 per acre. How much did I get for it?

7. Find the cost of 759 articles costing $5.77 each.

8. Find the cost of 367 things at $23.34 each.

9. There are 5280 feet in a mile. How many feet are there in 17 miles?

10. How many oranges are there in 47 boxes, if each box contains 189 oranges?

11. How many pounds of hay will 299 acres produce, if each acre produces 2178 pounds.

12. There are 4840 square yards in an acre. How many square yards in 127 acres?

13. If a state has an area of 8040 square miles, and a population of 279 to the square mile, what is the population of the state?

14. A merchant purchased 30 pieces of broadcloth, each containing 50 yards, at $8 per yard. How much did he pay for the whole?

15. I sold 27 loads of wheat, 35 bushels in each, at 93 cents a bushel. How much money did I receive?

ORAL REVIEW.

1. What is the cost of 2 oranges at 5 cents each and 4 bananas at 3 cents each?

2. I bought 2 slates at 15 cents apiece and 3 pencils at 4 cents each. How much did I pay for all?

3. I bought 3 pencils at 4 cents each and 4 oranges at 5 cents each. How much did I pay for all?

4. If a man earns $6 per week and a boy earns $3 per week, how much more will the man earn in 12 weeks than the boy?

5. A farmer has 30 bags of wheat, each containing 3 bushels. How much is the wheat worth at $1.00 per bushel?

6. If 6 teams can plough a field in 4 days, how long will it take one team to do the same work?

7. William has 25 cents and Harry has 4 times as many plus 10. How many has Harry?

8. A man bought a plough for $13 and 3 harrows at $9 apiece. How much did he pay for all?

9. How much must I pay for 3 books at 30 cents each and 5 at 25 cents each?

10. A matron purchased 4 bunches of asparagus at 10 cents a bunch and 5 quarts of strawberries at 13 cents each. How much did she pay for all?

11. George earned 95 cents a day; his board cost him 55 cents a day. How much did he save in 20 days?

12. A gallon contains 4 quarts, and a quart 2 pints. How many pints in 6 gallons?

13. A drover bought 50 sheep for 120 dollars; he sold 20 of them at 5 dollars a head and the rest at 4 dollars a head. How much did he gain?

14. I bought 4 quarts of milk at 8 cents a quart and 4 pounds of biscuits at 10 cents a pound. If I gave a dollar note in payment, how much should I receive back?

15. In an orchard are 26 peach-trees and 6 times as many apple-trees as peach-trees. How many trees in the orchard?

16. At 4.5 cents a pound, what will 320 pounds of sugar cost?

17. If 20 pennyweight make an ounce, what will a 2-ounce gold chain cost at $1.25 a pennyweight?

18. How much are 5 loads of flour worth, each load containing 12 barrels, at $6.00 a barrel?

19. A farmer's wife took to a store 6 pounds of butter at 30 cents a pound and bought 15 yards of calico at 10 cents a yard. Find the balance due her?

20. Mr. Green owns three houses; the first is worth $12,000, the second is worth twice as much as the first, and the third as much as both the first and second. How much is each house worth, and how much are they all together worth?

WRITTEN REVIEW.

1. Add 6 thousand 4 hundred 80, 4 thousand 4, 8 thousand 7, 8 hundred 89, 54 hundred 11.

2. A windmill pumps into a tank 25 gallons of water in an hour. After pumping 13 hours, how much does the tank, holding 500 gallons, lack of being filled?

3. A merchant bought 240 barrels of flour for $1920 and sold it at $10.50 a barrel. What did he gain?

4. Which cost the more and how much,—48 horses at $173.40 each or 1130 sheep at $4.60 a head?

5. If 169 tons of steel rails are required for a mile of railroad, how many tons are required for 449 miles?

6. A farmer bought 40 sheep at $7.00 each, 40 cows at $30 per head, and 4 horses at $200 each; he sold them all for $3000. How much did he gain?

7. What cost 37,560 tons of iron at $57 per ton?

8. What cost 293 barrels of flour at $7 per barrel, and 729 barrels at $8 per barrel?

9. Of a mill worth $78,000, B. owned $2365, C. owned $3600, and A. owned the remainder. What was A.'s share worth?

10. I paid $65 for a harness, $147 for a carriage, and for a horse as much as for the carriage and harness. What was the cost of all?

11. What is the difference between a million and a thousand?

12. What is the difference between a hundred million and a hundred thousand?

13. 25 masons earn $13 each and 20 laborers $8 each per week. How many dollars do they all earn in 12 weeks?

14. How many days in a year? If a man's income is $3000 a year and his daily expenses average $7.68, what does he save in a year?

15. A merchant bought a piece of broadcloth containing 56 yards for $133, and sold it at $3.00 a yard. How much did he make.

16. A gentleman paid $2500 for a driving team and

carriage, the team cost $380 more than the carriage. What was the value of the horses?

17. A speculator bought 140 acres of land for $7560 and sold 86 acres of it at $75 an acre, and the remainder at cost. How much did he make?

18. Find the cost in dollars and cents of 208 pounds of coffee at 28 cents per pound.

19. An orchard has 16 rows of apple-trees, and each row has 27 trees in it. If 30 bushels are gathered from each tree, how many bushels will the orchard produce?

20. Two men are 950 miles apart; they travel towards each other, one at the rate of 30 miles per day, and the other at the rate of 42 miles. At the end of 8 days, how far will they be apart?

21. A merchant's profits from his business were $8695. If he paid $869 for house rent, and three times as much for other expenses, how much did he save?

22. A drover bought 106 oxen, at $35 a head; it cost him $6 a head to get them to market, where he sold them at $47. Did he gain or lose, and how much?

23. A flour merchant bought 936 barrels of flour at $9 a barrel. He sold 480 of them at $9.50 a barrel, and the remainder at $8.50. What was the profit or the loss to him?

24. Two vessels start from the same port, and travel in opposite directions—one at the rate of 75 miles a day, the other at the rate of 85 miles a day. How far apart will they be at the end of 15 days?

25. Find the value of:

1. $99 \times 8 + 51 \times 10 - 7 \times 104 + 26$.

Suggestion: Multiply before adding or subtracting.

2. $56 + 17 + 13 \times 4 - 5 \times 15 + 8 \times 81$.
3. $3976 \times 23 + 3456 \times 25 - 2879 \times 34$.
4. $\$10.11 \times 924 - \$10.10 \times 777 + \$2.48 \times 123$.

DIVISION.

INDUCTIVE STEPS.

1. Since $2 + 2$ or two times $2 = 4$, how many 2's are there in 4?

2. Since $3 + 3 + 3$ or three times $3 = 9$, how many 3's are there in 9?

3. How many 4's are there in eight? In 12? In 20?

4. Then 4 is contained how many times in 8? In 12? In 20?

5. How many times is:
   1. \$3 contained in \$6? \$4 contained in \$12? \$5 in \$20?
   2. 6 books contained in 18 books? 7 gallons contained in 21 gallons?

The process by which we determine that 7 is contained in 21 three times is called **Dividing**.

6. Divide:
   1. 10 quarts by 2 quarts. 10 quarts by 5 quarts.
   2. 18 horses by 3 horses. 30 men by 10 men.
   3. 45 pounds by 9 pounds. 12 weeks by 6 weeks.

DEFINITIONS.

1. **Division** is the process of finding how many times one number is contained in another, or of finding one of the equal parts into which a number is to be separated.

The latter part of this definition is discussed on page 115, under the head of Equal Parts.

2. The **Dividend** is the number to be divided.

3. The **Divisor** is the number used in effecting a division.

4. The **Quotient** is the result of a division, and answers the question, How many times? or, What is one of the equal parts?

5. The **Remainder** is the part of the dividend left after an incomplete division.

6. The **Signs** of division are $\div$, ——, and). Each one is read "divided by."

7. The dividend is placed: On the left of $\div$, as in $12 \div 4 = 3$; above ——, as in $\frac{12}{4} = 3$; on the right of), as in $4\overline{)12}$, or $4)12(3$.

$$\begin{array}{r} 3 \\ \hline 0 \end{array} \qquad \begin{array}{r} 12 \\ \hline \end{array}$$

Each of these equations is read, "12 divided by 4 equals 3."

Equal Parts.

1. When anything is divided into two equal parts, is not each part called *one-half* of that thing?

2. What is *one-half* of 4? Of 8? Of 10? Of 12?

3. Write the division of 4, 8, 10, and 12 by 2 in each of the ways explained in 7.

4. When anything is divided into 3 equal parts, can you tell what each part is called?

5. What is *one-third* of 6? Of 9? Of 12? Of 15?

6. By means of signs express the division of 6, 9, 12, and 15, by 3.

7. We see that the divisor gives the name to the equal parts.

1. Divisor 2 names each equal part *one-half*, written $\frac{1}{2}$.
2. Divisor 3 names each equal part *one-third*, written $\frac{1}{3}$.
3. Divisor 4 names each equal part *one-fourth*, written $\frac{1}{4}$.
4. Divisor 5 names each equal part *one-fifth*, written $\frac{1}{5}$.
5. Divisor 9 names each equal part *one-ninth*, written $\frac{1}{9}$.

8. Dividends denote the number of equal parts, as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$, read "*one*-half, *two*-thirds, *four*-fifths."

9. Equal parts thus indicated are called **Fractions.**

ORAL EXERCISES.

1. In the fraction $\frac{2}{3}$, point out the dividend and divisor.

2. In the fraction $\frac{12}{6}$, point out dividend and divisor, and name the quotient.

3. To find the quotient, what part of 12 did you take?

4. How, then, do you find $\frac{1}{6}$ of a number? One-ninth? $\frac{1}{12}$? One-twentieth?

5. Read the following fractions:

1. $\frac{4}{5}$, $\frac{6}{7}$, $\frac{5}{10}$, $\frac{6}{13}$, $\frac{3}{8}$, $\frac{5}{9}$, $\frac{4}{11}$, $\frac{7}{12}$, $\frac{8}{15}$, $\frac{2}{17}$, $\frac{3}{19}$, $\frac{4}{21}$.
2. $\frac{4}{19}$, $\frac{5}{21}$, $\frac{6}{49}$, $\frac{7}{23}$, $\frac{8}{30}$, $\frac{9}{40}$, $\frac{10}{50}$, $\frac{19}{100}$, $\frac{21}{200}$, $\frac{41}{250}$.
3. $\frac{11}{12}$, $\frac{21}{33}$, $\frac{32}{45}$, $\frac{48}{108}$, $\frac{57}{95}$, $\frac{64}{1000}$, $\frac{75}{1200}$, $\frac{144}{1728}$, $\frac{231}{1900}$.

6. What is the quotient of:

1. $9 \div 3$?	6. $16 \div 4$?	11. $28 \div 4$?
2. $36 \div 6$?	7. $49 \div 7$?	12. $28 \div 7$?
3. $63 \div 7$?	8. $45 \div 5$?	13. $27 \div 9$?
4. $40 \div 4$?	9. $56 \div 7$?	14. $54 \div 9$?
5. $15 \div 5$?	10. $21 \div 3$?	15. $42 \div 6$?

7. What is the quotient of:

1. $\frac{36}{6}$?	3. $\frac{30}{5}$?	5. $\frac{66}{11}$?	7. $\frac{72}{8}$?
2. $\frac{72}{9}$?	4. $\frac{100}{10}$?	6. $\frac{108}{12}$?	8. $\frac{24}{8}$?

8. What is the quotient of:

(1.) $5\overline{)35}$?	(2.) $5\overline{)25}$?	(3.) $8\overline{)32}$?	(4.) $9\overline{)36}$?
(5.) $7\overline{)63}$?	(6.) $11\overline{)88}$?	(7.) $8\overline{)56}$?	(8.) $11\overline{)132}$?
(9.) $2\overline{)24}$?	(10.) $3\overline{)36}$?	(11.) $4\overline{)44}$?	(12.) $5\overline{)60}$?
(13.) $6\overline{)66}$?	(14.) $7\overline{)63}$?	(15.) $8\overline{)96}$?	(16.) $12\overline{)144}$?

ORAL PROBLEMS.

1. How many pounds of sugar, at 5 cents a pound, can be bought for 35 cents.

SOLUTION.—Since 1 pound costs 5 cents, for 35 cents as many pounds can be bought as 5 cents is contained *times* in 35 cents, or 7 pounds.

When the quotient answers How many times, are not the dividend and divisor like numbers?

2. If a man earns $3 per day, how long will it take him to earn $12?

3. When eggs are selling for 12 cents a dozen, how many dozen will 60 cents buy?

4. For 21 cents how many pens can be bought at 3 cents each?

5. How many times can 5 gallons be taken from a cask containing 30 gallons?

6. If a man can dig 7 rods of ditch in a day, how many days will it take him to dig 28 rods?

7. If a man pays \$56 for seven yards of cloth, how much is that a yard?

SOLUTION.—Since 7 yards cost \$56, one yard costs $\frac{1}{7}$ of \$56, or \$8.

By what did you divide to take $\frac{1}{7}$ of 56?

Did you divide by 7 yards or by the abstract number 7?

8. How much hay, at \$8 per ton, can be bought for \$32?

9. If \$55 is paid for 11 cords of wood, what is the price per cord?

10. If a farm of 120 acres is divided into 12 equal lots, how many acres does each lot contain?

11. There are 96 trees in an orchard, and 12 trees in each row. How many rows are there?

12. In 12 hours 108 horses crossed a bridge. What was the average number per hour?

13. A man who had 48 acres of land divided it into 6 fields of equal size. What part of the whole was each field? How many acres were in each field?

14. If I deposit \$12 in a savings bank every month, in how many months shall I deposit \$132?

15. If 6 car-loads of freight weigh 18 thousand pounds, how much does each car-load weigh?

16. A merchant paid 144 cents for an advertisement of 12 lines. What was the rate per line?

17. If 6 pairs of boots cost 54 dollars, what will 10 pairs cost?

ANALYSIS.—If 6 pairs cost 54 dollars, 1 pair costs $\frac{1}{6}$ of 54 dollars, or 9 dollars, and 10 pairs cost 10 *times* 9 dollars, or 90 dollars.

18. In one week there are 7 days. How many weeks are there in 25 days, and how many days remain?

19. Referring to the previous example, 7 times 3, plus 4, equals what?

PRINCIPLE.

1. When the dividend and divisor are like numbers, the quotient is an abstract number denoting "times."

2. When the divisor is an abstract number, the dividend and quotient are like numbers, the quotient being a part of the dividend.

3. The product of the divisor and quotient, plus the remainder, equals the dividend.

The Divisor a Single Figure.

WRITTEN EXERCISES.

1. Divide 1728 by 4.

Process.

Divisor. Dividend. Quotient.

$$4\overline{)1728}\,(432$$

16

12

12

8

8

0

Explanation.

Beginning at the left, we perceive that 4 will not divide 1, but 1 thousand + 7 hundreds = 17 hundreds; 17 hundreds ÷ by 4 = 4 hundreds, with 1 hundred remaining; 1 hundred + 2 tens = 12 tens; 12 tens ÷ by 4 = 3 tens, with no tens remaining; the 8 units ÷ by 4 = 2 units. Hence, the quotient is 432, with 0 remaining.

Proof.

$432 \times 4 = 1728$.

The preceding process is called **Long Division.**

A second process, called **Short Division,** is as follows:

Process.

$$4\overline{)1728}$$

432

Explanation.

After dividing 17 by 4, the one hundred remaining, instead of being written, is prefixed mentally to the 2 tens, making 12 tens.

2. Solve both by long and short division the following:

1. 963 ÷ 3.	11. 2450 ÷ 2.	21. $49.86 ÷ 9.
2. 1926 ÷ 6.	12. 3252 ÷ 3.	22. $67.65 ÷ 5.
3. 3672 ÷ 4.	13. 4872 ÷ 4.	23. $38.36 ÷ 7.
4. 3246 ÷ 6.	14. 6830 ÷ 5.	24. $98.72 ÷ 8.
5. 2763 ÷ 9.	15. 2976 ÷ 6.	25. $45.60 ÷ 5.
6. 3235 ÷ 5.	16. 2985 ÷ 5.	26. $83.34 ÷ 6.
7. 2205 ÷ 7.	17. 4635 ÷ 3.	27. $81.72 ÷ 9.
8. 2696 ÷ 8.	18. 3986 ÷ 4.	28. $91.84 ÷ 4.
9. 3456 ÷ 9.	19. 3248 ÷ 8.	29. $26.64 ÷ 3.
10. 2465 ÷ 5.	20. 5256 ÷ 6.	30. $27.37 ÷ 7.

3. Divide 607 by 2. Also, 617 by 2.

Process.	Process.	Explanation.
2)607	2)617(308	In the first case 6 ÷ 2 = 3; 0 ÷ 2 = 0; 7 ÷ 2 = 3, with 1 remaining.
303 — 1 rem.	6	
	17	In the second case 6 ÷ 2 = 3; 1 brought down and divided by 2 = 0; 1 ten and the 7 units brought down = 17 units; 17 units divided by 2 = 8 units, with 1 unit remaining.
	16	
	1 rem.	

Prove by the principle.

4. Find the quotients and remainders of:

1. 737 ÷ 2.	10. 4055 ÷ 6.	19. 48,743 ÷ 2.
2. 736 ÷ 3.	11. 9767 ÷ 8.	20. 53,742 ÷ 3.
3. 963 ÷ 4.	12. 9896 ÷ 5.	21. 65,889 ÷ 4.
4. 916 ÷ 5.	13. 9894 ÷ 9.	22. 47,397 ÷ 5.
5. 824 ÷ 6.	14. 9568 ÷ 3.	23. 67,327 ÷ 7.
6. 937 ÷ 7.	15. 9830 ÷ 8.	24. 49,538 ÷ 6.
7. 995 ÷ 8.	16. 9847 ÷ 7.	25. 93,570 ÷ 7.
8. 965 ÷ 9.	17. 7408 ÷ 7.	26. 96,674 ÷ 8.
9. 679 ÷ 8.	18. 3943 ÷ 4.	27. 57,487 ÷ 9.

28. 986 ÷ 5.	39. 4792 ÷ 9.	50. 38,596 ÷ 3.
29. 927 ÷ 6.	40. 4765 ÷ 9.	51. 98,696 ÷ 5.
30. 965 ÷ 7.	41. 4827 ÷ 4.	52. 96,575 ÷ 7.
31. 995 ÷ 4.	42. 5672 ÷ 6.	53. 94,701 ÷ 8.
32. 503 ÷ 7.	43. 7484 ÷ 6.	54. 69,886 ÷ 9.
33. 765 ÷ 2.	44. 9832 ÷ 7.	55. 86,340 ÷ 3.
34. 998 ÷ 8.	45. 9323 ÷ 9.	56. 90,678 ÷ 4.
35. 963 ÷ 5.	46. 9524 ÷ 6.	57. 99,365 ÷ 6.
36. 965 ÷ 9.	47. 6498 ÷ 4.	58. 93,720 ÷ 3.
37. 957 ÷ 4.	48. 8110 ÷ 7.	59. 94,831 ÷ 8.
38. 974 ÷ 6.	49. 1474 ÷ 6.	60. 95,942 ÷ 9.

With the divisor and remainder of each of the above examples make a fraction; thus (1) $\frac{1}{2}$, (2) $\frac{1}{3}$, (3) $\frac{3}{4}$, etc.

Write the exact quotient of all the examples in the second column; thus (21) $675\frac{5}{6}$.

WRITTEN PROBLEMS.

1. How many 6-pound packages of buckwheat flour can be made from 1200 pounds of flour?

2. How many ploughs at $7 each can be bought for $1232?

3. How many tons of coal at $7 a ton can be bought for $1995?

4. There are 3 feet in one yard. How many yards are there in 63,360 feet?

5. There are 8 quarts in a peck. How many pecks in 525,232 quarts?

6. 4 pecks make 1 bushel. How many bushels do 249,024 pecks make?

7. If 9 cents will buy one pound of cotton, how many pounds will 507,285 cents buy?

8. If \$4 will buy one yard of velvet and \$1 will buy $\frac{1}{4}$ of a yard, how many yards and fourths of a yard can be bought for \$23?

9. How many times must you take \$7 to make \$1134? How many times must you take \$9?

10. A stage travelled 8 miles per hour. In how many hours would it travel, at the same rate, 4704 miles?

11. A bicycler rides at the rate of 12 miles per hour. How long would it take him to ride 1728 miles?

12. James Blair paid \$37,504 for some Western land at \$8 per acre. How many acres did he buy?

13. A man gave \$36,755 in equal shares to his 5 children. How much did he give to each child?

14. If sound moves 10,080 feet in 9 seconds, how many feet does it move in 1 second?

15. 9 square feet make 1 square yard. How many square yards in 43,560 square feet?

16. 7 days equal 1 week. In 364 days how many weeks?

17. How many yards of cloth, at 9 cents per yard, can be bought for 58,878 cents?

18. How many \$5 bills must be counted out to pay a bill of \$1240.

19. How many pounds of sugar, at 8 cents a pound, must be given for 488 pounds of coffee, at 22 cents a pound?

20. Find the value of 240 bottles of wine at \$3 a bottle, and find how many \$4 bottles of wine must be given in exchange for the \$3 bottles.

21. Prove that if we multiply a given number by 100

and divide the product by 4 we obtain the same result as when multiplying by 25.

22. Prove by the asterisks in the margin that $4 \times 5 = 5 \times 4$.

The Divisor a Single Digit with Ciphers Annexed.

ORAL EXERCISES.

1. If 50 cents be put in 10 equal packages, how many cents will each package contain?

2. When flour is worth $10 a barrel, how many barrels can be bought for $110?

3. 120 divided by 10 gives what quotient? What remainder? Does not cutting off the cipher, thus 12/0, give you the same results?

4. How many times is 10 contained in 124? What remains? Does not cutting off the 4 thus, 12/4, give the same results? What is the exact quotient?

5. How, by cutting off, can you divide 124 by 100? What is the quotient? What is the remainder? What is the exact quotient?

6. How many figures will you cut off to divide by 1000?

PRINCIPLES.

1. **Cutting off one figure divides by 10.**
2. **Cutting off two figures divides by 100.**
3. **Cutting off three figures divides by 1000, and so on.**

WRITTEN EXERCISES.

1. Divide 365 by 10.

Process.

Divisor. Dividend.

$10\,)\,36|5$

$36\frac{5}{10}$, Quotient.

Explanation.

In accordance with Principle 1, to divide by 10, we cut off one figure from the dividend, and obtain for quotient 36, with 5 as remainder. Hence, the exact quotient is $36\frac{5}{10}$.

2. Divide 1728 by 300.

Process.

$3|00\,)\,17|28$

$5\frac{228}{300}$.

Explanation.

In accordance with Principle 2, to divide by 100 we cut off two figures, and obtain for quotient 17, with 28 remaining. Since the entire divisor is 300, we must divide this quotient by three. One-third of 17 hundreds = 5 hundreds, with two hundreds remaining; 2 hundreds = 200 units; 200 units plus 28 units = 228 units. Hence, the exact quotient is $5\frac{228}{300}$.

Hence, the following rule:

1. **Cut off the ciphers from the divisor.**
2. **Cut off as many figures from the dividend.**
3. **Divide by the significant figure of the divisor.**
4. **Make a fraction with the remainder and divisor.**

To apply the rule:

3. Divide 1898 by 400.

Process.

$4|00\,)\,18|98$

$4\frac{298}{400}$

Explanation.

Having cut off the two ciphers from 400 and two figures from the dividend, we obtain 18 for quotient and 98 for remainder. Dividing now by the significant figure 4, we obtain 4 for quotient and 200 + 98 for remainder. Making a fraction with the remainder and divisor, we have for the exact quotient $4\frac{298}{400}$.

4. Divide:

1. 89 by 10.	16. 96,704 by 3000.
2. 376 by 100.	17. 54,970 by 4000.
3. 3422 by 100.	18. 49,685 by 5000.
4. 5489 by 1000.	19. 74,769 by 6000.
5. 6079 by 1000.	20. 82,546 by 7000.
6. 71,560 by 100.	21. 99,839 by 8000.
7. 97,048 by 100.	22. 99,953 by 9000.
8. 57,084 by 1000.	23. 123,456 by 1000.
9. 97,068 by 1000.	24. 789,012 by 2000.
10. 95,650 by 1000.	25. 345,678 by 3000.
11. 7936 by 40.	26. 9,012,345 by 40,000.
12. 3079 by 50.	27. 6,789,012 by 50,000.
13. 4987 by 300.	28. 3,456,789 by 60,000.
14. 5097 by 500.	29. 1,982,734 by 70,000.
15. 90,798 by 2000.	30. 5,678,901 by 80,000.

The Divisor Any Number of Digits.

EXERCISES.

1. Divide 25,003 by 48.

Process.

$$48\overline{)25003}\,(520\tfrac{43}{48}$$

240

100

96

43

Explanation.

Beginning at the left, we perceive that 48 will divide neither 2 nor 25. We therefore divide 48 into 250 hundreds, and obtain 5 hundreds for quotient. 48 × 5 hundreds = 240 hundreds; 250 hundreds — 240 hundreds = 10 hundreds. Annexing 0 tens from the dividend, we have 100 tens; dividing 100 tens by 48, we obtain 2 tens. 48 × 2 tens = 96 tens; 100 tens — 96 tens = 4 tens. Annexing 3 units from the dividend, we have 43; 48 is contained in 43 no times, and we write 0 in the quotient. 43 is the remainder, and the exact quotient is $520\frac{43}{48}$.

Proof.

$520 \times 48 + 43 = 25{,}003.$

Help Table.

$48 \times 0 = 0$	$48 \times 5 = 240$
$48 \times 1 = 48$	$48 \times 6 = 288$
$48 \times 2 = 96$	$48 \times 7 = 336$
$48 \times 3 = 144$	$48 \times 8 = 384$
$48 \times 4 = 192$	$48 \times 9 = 432$

What method was employed above, Long Division or Short Division?

Look at the table and state why neither 4 nor 6 can be taken for the first figure of the quotient.

RULE.

1. Begin at the left to divide.

2. Divide the divisor into the fewest figures that will contain it.

3. Multiply the divisor by the quotient figure.

4. Subtract, and annex the next figure of the dividend.

5. Divide the divisor into the number thus formed.

6. Multiply, subtract, and annex the next figure of the dividend.

7. So proceed until all the figures of the dividend have been used.

8. Finally, make an exact quotient, if necessary.

2. Divide 304,675 by 48, using the above table.

3. Divide 789,101 by 56, first having made a table to aid you in this work.

4. Divide 123,456 by 27.

5. Divide:

1. 7890 by 10.
2. 5303 by 11.
3. 3667 by 12.
4. 7590 by 13.
5. 4089 by 14.
6. 9085 by 15.
7. 2476 by 16.
8. 3420 by 17.
9. 2599 by 18.
10. 3858 by 19.
11. 4420 by 20.
12. 4605 by 21.
13. 4807 by 23.
14. 4602 by 34.
15. 6095 by 45.
16. 4555 by 56.
17. 4478 by 67.
18. 4135 by 78.
19. 3708 by 89.
20. 7334 by 99.
21. 8178 by 73.
22. 4952 by 93.
23. 6840 by 82.
24. 5198 by 71.
25. 7716 by 60.
26. 8840 by 40.
27. 9210 by 42.
28. 9614 by 46.
29. 9204 by 68.
30. 9110 by 57.
31. 15,780 by 15.
32. 15,909 by 16.
33. 14,668 by 17.
34. 37,950 by 18.
35. 24,534 by 19.
36. 63,595 by 20.
37. 17,332 by 30.
38. 27,360 by 50.
39. 20,792 by 60.
40. 34,652 by 70.
41. 44,200 by 80.
42. 46,050 by 90.
43. 14,421 by 100.
44. 13,806 by 101.
45. 24,380 by 102.
46. 22,775 by 121.
47. 26,868 by 123.
48. 28,945 by 130.
49. 29,664 by 144.
50. 66,006 by 156.
51. 81,780 by 160.
52. 54,472 by 172.
53. 82,080 by 185.
54. 51,980 by 190.
55. 17,716 by 200.
56. 17,680 by 205.
57. 27,630 by 221.
58. 38,456 by 276.
59. 46,020 by 290.
60. 54,660 by 223.

6. Divide:

1. 300,165 by 327.
2. 586,635 by 404.
3. 236,823 by 526.
4. 304,727 by 783.
5. 343,926 by 250.
6. 316,743 by 684.
7. 202,729 by 672.
8. 349,052 by 879.
9. 594,924 by 709.
10. 458,015 by 532.
11. 320,479 by 373.
12. 329,500 by 402.
13. 660,209 by 723.
14. 2,658,360 by 3245.
15. 3,360,843 by 5023.
16. 4,300,436 by 6436.
17. 2,010,081 by 7292.
18. 3,453,901 by 6007.
19. 2,980,506 by 7035.
20. 4,007,205 by 6294.
21. 1,795,791 by 7272.
22. 9,500,367 by 5959.
23. 7,989,368 by 7979.
24. 1,509,307 by 5757.
25. 8,640,073 by 8383.
26. 4,560,079 by 4949.
27. 9,200,537 by 8787.
28. 6,975,841 by 9696.
29. 7,935,836 by 8484.
30. 5,009,703 by 7373.
31. 9,071,564 by 6969.
32. 3,795,411 by 8383.

ORAL PROBLEMS.

1. Hats are $4 apiece; broadcloth is $6 per yard. How many of the hats will pay for 6 yards of the broadcloth?

2. Twelve lambs cost $24. To gain $12, how must they be sold per head?

3. There are 3 feet in a yard. How many yards in 627 feet?

4. A boy earns $3.00 per week. After paying $1.00 per week for other things, how long will it take him to earn enough to buy a suit of clothes costing $16.00?

5. A man had $60; he spent one-half of it for sheep at $3.00 each. How many sheep did he buy?

6. A train goes 40 miles per hour. How long will it take to go 480 miles?

7. A newsboy bought 10 papers for 35 cents and sold them so as to gain 15 cents. How much did he get apiece for them?

8. I bought 10 tons of coal for $50 and sold it so as to gain $20. How much did I gain per ton?

9. A lady having $5 bought 5 chickens at 50 cents each and a turkey for $1.50. How much money had she left?

10. What would be the cost of 1 arithmetic if $162.96 were paid for 400 copies?

11. If a farmer exchanges 6 firkins of butter, worth $20 a firkin, for cloth at $4 a yard, how many yards will he receive?

12. At $5 per gallon, how much alcohol can be bought for $37.

WRITTEN PROBLEMS.

1. A dividend is 14,145, and the quotient 123. What is the divisor? State the principle.

2. A certain number is contained 41 times in 1043, with 18 as a remainder. What is the number?

3. Paid $3780 for 28 acres of land. How much was that per acre?

4. How many bushels of corn at 56 cents each can be bought for 7560 cents.

5. A man raised 6427 bushels of oats upon 107 acres. What was the average crop per acre?

6. If a farmer can lay up $425 a year, how many years will it take him to lay up $6800?

7. A field of 600 acres produced 8400 bags of wheat. How many bags to the acre was that?

8. A man paid $170,352 for 36 city lots. What was the average cost per lot?

9. In 94,185 yards of sheeting are how many pieces, each piece containing 45 yards?

10. A surveyor travels 41,600 rods in one week. There being 320 rods in a mile, how many miles does he travel?

11. The distance of the earth from the sun is 91,500,000 miles. Light travels at the rate of 185,000 miles per second. In how many seconds does light travel from the sun to the earth?

12. At the rate of $75 per acre, how many acres of land can be bought for 40,425 dollars?

13. A man bought 133 horses and 160 mules, paying 39,555 dollars for the whole. What was the average price?

14. The divisor is 88, the quotient $248, and the remainder $66. What is the dividend?

15. The product of two numbers is 40,796. One of the numbers is 124. What is the other number?

16. How many times must a pole 11 feet long be applied to measure a distance of a mile (5280 feet)?

17. A long ton of coal is 2240 pounds and a short ton is 2000 pounds. Find how many short tons weigh as much as 250 long tons.

18. A man has 8000 dollars. He buys two houses for 4500 dollars, and some land at 140 dollars per acre. How many acres will he get?

19. Bought 288 barrels of flour at $1482, and sold the same for $2058. What was the gain on each barrel?

20. How many horses at $82.50 each, can be bought for $6187.50.

21. There are 7000 Troy grains in a pound avoirdupois. How many pounds are there in 118,125 grains.

22. 3600 seconds make an hour. How many hours in 12,950 seconds.

23. How many schooners, each carrying 8700 bushels of wheat, will be required to carry 843,900 bushels?

24. How long can 125 men subsist on an amount of food that will last 1 man 4500 days?

Parenthesis and Vinculum.

1. **A Parenthesis,** ( ), signifies that the numbers within it are to be subjected to the same operation, and in consequence the operations indicated within it should first be performed.

In the expression, $(16 - 4) \div 4$, we first subtract 4 from 16, and then divide the result by 4.

2. **A Vinculum,** ———, may be used instead of the parenthesis.

$\overline{6 - 4} \times 4$ and $(6 - 4) \times 4$ have precisely the same meaning.

EXERCISES.

Find the value of:

1. $(423 + 47) - (492 - 326) - 76$.
2. $(325 - 92) - (226 - 29 + 7) + 20$.
3. $(413 - 200) - (118 - \overline{24 - 4} + 6) + 3$.
4. $(282 - 97) - (\overline{4 \times 5} + 38) + 20$.
5. $(4 \times 5 \times 9) - (\overline{3 + 9} \div 4 + 6)$.
6. $(6 + 2 + 7) \times 5 - \overline{(8 + 9 + 4) \div 7} + 20$.
7. $(23 + 8 - 1) \times 6$.

8. $24 - 7 + 18 \times 7$.
9. $(22 - 3 + 26) \times 9$.
10. $(3 + 4) \times 9 - (3 + 6) \div 3$.
11. $\overline{(6 + 8 - 4) \times 4} + \overline{(203 + 67) \div 3}$.
12. $\overline{(100 - 1) \div 9} - \overline{(97 + 20) \div 13} + \overline{(4 + 8 \div 4}$.
13. $(56 + 4) \div 6 + (21 + 24) \div (8 - 3) + 7$.
14. $(105 \div 21) + (80 \div 5) \times (81 + 36 \div 9)$.
15. $327 \times 6 \div 109 + 52 \times 5 - (42 + 8 \times 4)$.

ANALYSIS.

Analysis reasons from the given number *to one*, and then *from one* to the required number.

PROBLEMS.

1. If 7 barrels of flour cost 35 dollars, what will 9 barrels cost?

Analysis.	**Explanation.**
7 barrels = 35 dollars. 1 barrel = 5 dollars. 9 barrels = 45 dollars.	If 7 barrels cost 35 dollars, 1 barrel costs $\frac{1}{7}$ of 35 dollars, or 5 dollars; if 1 barrel costs 5 dollars, 9 barrels cost 9 times 5 dollars, or 45 dollars.

2. If 7 men can do a piece of work in 8 days, in how many days can 8 men do it?

Analysis.	**Explanation.**
7 men = 8 days. 1 man = 56 days. 8 men = 7 days.	It will take 1 man longer than 7 men to do a piece of work; hence, we say "If 7 men can do the work in 8 days, 1 man will do it in 7 times 8 days, or 56 days; and if 1 man can do it in 56 days, 8 men can do it in $\frac{1}{8}$ of 56 days, or 7 days."

3. A man bought sheep at the rate of 3 for \$18. How many did he buy for \$906?

Analysis.	Explanation.
3 sheep = \$18. 1 sheep = \$6. \$906 ÷ \$6 = 151 sheep.	If 3 sheep cost \$18, 1 sheep costs $\frac{1}{3}$ of \$18, or \$6; if 1 sheep costs \$6, for \$906 he bought as many sheep as \$6 is contained *times* in \$906, or 151 sheep.

4. In 17 miles there are 29,920 yards. How many yards are there in 35 miles?

5. If 12 barrels of flour are worth \$120, what will 36 barrels cost?

6. If 45 acres produce 2520 bushels of corn, how many bushels do 28 acres produce at the same rate?

7. A merchant buys 168 barrels of flour for \$672. How many barrels can he buy for \$984 at the same rate?

8. If a man can travel 154 miles in 7 days, how far can he travel in 43 days?

9. If a stack of oats will serve 25 horses 9 days, how many days will it serve 15 horses?

10. If 8 men can do a piece of work in 145 days, how long will it take 29 men to do the same work?

11. If 46 acres of land produce 2484 bushels of corn, how many bushels will 120 acres produce?

12. At 6 cents each, how many oranges can be bought for \$5.58?

13. If 29 tons of coal cost \$116, what will 37 tons cost?

14. What is the cost of 68 barrels of molasses if 37 barrels cost \$888?

15. A farmer sold 18 calves at the rate of 3 for \$33. How much did he get for them?

16. If 19 horses can be bought for 1520 dollars, how many horses can be bought for 1360 dollars?

17. How long will it take 19 men to do a piece of work that 17 men can do in 133 days?

18. 25 barrels of flour weigh 4900 pounds. Find the weight of 87 barrels.

19. If 36 men can be hired for $50.40 for one day, how many men can be hired for the same time for $133?

20. If I pay $96 for 25 hats, how much must I pay for 63 hats at the same rate?

21. A certain quantity of barley lasts 11 horses 15 days. How long would it last 5 horses?

22. I bought 9 horses for $1530, and sold them for $1665. How much did I gain on each horse?

23. If 13 hogsheads of molasses can be bought for 273 dollars, how many hogsheads can be bought for 609 dollars?

24. A man bought 150 calves at $14 a head and sold them so as to gain $450. What did he get a head?

25. In how many weeks can a father and son together earn $71.75, if the father earns $10.60 and the son $3.75 per week?

26. What cost 7 pieces of muslin, 37 yards each, at $.13 per yard?

27. Engaged in farming, I wish to obtain $370; I therefore sell 100 bushels of wheat at $0.75 per bushel and enough apples at $2.50 per barrel to obtain the sum required. How many barrels of apples do I sell?

28. A man bought 320 barrels of apples at $3 per barrel. He found 120 barrels worthless. At what rate must he sell the rest to get back all the money expended?

29. If a merchant buys coal at the rate of $3.75, and

sells it at $5.00 per ton, how many tons must he sell in order to gain $1500?

30. A grocer bought 250 pounds of coffee for $82.50, and sold it at 37 cents a pound. What was his gain?

31. A year contains 365 days. The yearly salary of the President of the United States is 50,000 dollars. How much can he spend daily and save at the end of the year 9850 dollars?

32. If 16 horses cost $1952, what will 22 horses cost at $6 less a head?

33. If 6 dollars' worth of flour lasts a family of 8 persons 2 months, how many dollars' worth will last 13 persons the same length of time?

Suggestion: 8 persons = 6.00 dollars; 1 person = .75 dollars; 13 persons = how many dollars?

34. If a man could reap a field of oats in 16 days by working 12 hours a day, in how many days could he reap it by working 8 hours per day?

35. I sold land at $46 an acre, and received for it $4508. How much would I have received for it, had I sold it at $58 an acre?

36. A man bought 96 apples at the rate of 4 for 6 cents, and exchanged them for pears at 8 cents apiece. How many pears did he receive?

37. Mr. Moore, having $10,000, invests $3750 in a house, and the remainder in land at $125 an acre. How much land does he buy?

38. Find the value of $(152 + 119 - 118) \div 9$.

39. Find the value of $(96 - 84 \div 7) \times 10$.

40. The quotient is 367, the divisor is 445, and the remainder 189. What is the dividend?

41. In payment of a bill, a lady gave 30 twenty-dollar bills, 10 ten-dollar bills, 4 one-dollar bills, and 16 five-dollar bills. How large was the bill she paid?

42. A pupil being told to multiply a certain number by 23, mistook the 3 for a 5, and his answer was 150. What was the correct answer?

43. If 20 men can do a piece of work in 31 days, in how many days will the work be done, if 11 more men are employed?

44. Find the value of (XLV. + III.) ÷ VI. + (X. + XV.) + (VII. — II.) + VI.

REVIEW.

1. Define the following terms:

1. Division.	6. One-half.
2. Dividend.	7. One-third.
3. Divisor.	8. Parenthesis.
4. Quotient.	9. Vinculum.
5. Remainder.	10. Analysis.

2. Repeat from memory the first three principles of Division.

3. Repeat the next three principles and the rule, in four parts, derived therefrom.

4. Repeat the eight parts of the rule of long division.

5. Describe the signs of division.

6. Define, also, *Multiplication*, *Multiplicand*, *Multiplier*, *Product*, *Abstract Number*, *Denominate Number*.

7. Give the six principles of multiplication.

8. Define *Subtraction*, *Minuend*, *Subtrahend*, *Remainder*.

9. Repeat the principles and the rule of subtraction.

10. Repeat the principles and the rule of addition.

11. Define:

1. Unit.
2. Number.
3. Arithmetic.
4. Notation.
5. Numeration.
6. Analysis.
7. Decimal Point.
8. Arabic System.
9. Roman System.
10. Zero.

12. Repeat the principles of both the Arabic and Roman systems.

FACTORING.

INDUCTIVE STEPS.

1. $2 \times 5 = 10$, being an expression of equality, is called what? 2 and 5, as *makers* of 10, are called what?

2. Make an equation showing that 2 and 3 are factors of 6.

3. Will each of the factors 2 and 3 exactly divide 6?

4. Make an equation showing the product of two factors equal to 12.

5. If your equation is $3 \times 4 = 12$, will 3 and 4 exactly divide 12? Is not a factor, then, an exact divisor?

6. Has 3 any factors except itself and 1? Has 4 any factors except itself and 1?

7. Now make an equation, putting 3 factors equal to 12, and excluding 1 as a factor?

8. In the following numbers point out each that has no factors except itself and 1: *1*, *2*, *3*, *4*, *5*, *6*, *7*, *8*, *9*.

9. Point out all the factors of these: 4, 6, 8, 9.

Which one has the greatest number of factors? What *two* factors will produce 8?

10. What are the exact divisors of the following numbers: 16, 18, 20, 21, 43, 27, 29, 35, 42, 63, 17, 72, 37, 144.

It is to be specially noted that while 1 may be considered a factor of any number, there are certain numbers that can have no factors if 1 is excluded. It is also to be specially noted that by factors and divisors are meant only integral factors and divisors.

DEFINITIONS.

1. An **Integral Number** expresses whole units.

23 and 16 are integral numbers, or *integers*.

2. An **Exact Divisor** is an integer that divides a number without a remainder.

2, 4, 6, 9 are exact divisors of 36.

3. The **Factors** of a number are integers that, multiplied together, produce the number.

7 and 8 are factors of 56.

4. A **Prime Number** has no exact divisors except itself and 1.

1, 3, 5, 7, 11, 13 are prime numbers.

5. **Prime Factors** are prime numbers used as factors.

7 and 11 are prime factors of 77.

6. A **Composite Number** has other factors than itself and 1.

8, 9, 12, 16 are composite numbers.

7. An **Even Number** is exactly divisible by 2.

4, 10, 14, 22 are even numbers.

8. An **Odd Number** is not exactly divisible by 2.

1, 3, 5, 7, 9, 11 are odd numbers.

9. **Factoring** is the process of finding the factors of a number.

10. An **Exponent** is a figure employed to show how often a factor occurs in a given number.

$8 = 2 \times 2 \times 2 = 2^3$. 3 is the exponent.

PRINCIPLES.

1. **A factor of a number is an integer.**
2. **A factor of a number is an exact divisor of it.**
3. **Only a prime factor of a number, or the product of two or more of its prime factors, is an exact divisor of that number.**

WRITTEN EXERCISES.

1. Find the prime factors of 468.

Process.

$$\begin{array}{r|l} 2 & 468 \\ 2 & 234 \\ 3 & 117 \\ 3 & 39 \\ & 13 \end{array}$$

Explanation.

Since the prime factors of 468 are exact divisors of it, we may find its prime factors by finding the prime numbers that will divide it. Dividing, we find the prime factors of 468 to be 2, 2, 3, 3, 13, or 2^2, 3^2, 13.

Proof.

$2^2 \times 3^2 \times 13 = 468$.

RULE.

Using continuously the least prime number that will serve as a divisor, divide the given number, and the succeeding quotients until the quotient is a prime number. The divisors and the last quotient will be the prime factors.

2. Find the prime factors of:

1. 228	12. 216	23. 320	34. 1250
2. 324	13. 484	24. 500	35. 1024
3. 224	14. 576	25. 965	36. 1728
4. 344	15. 432	26. 719	37. 1280
5. 144	16. 672	27. 913	38. 1152
6. 225	17. 396	28. 745	39. 3204
7. 796	18. 625	29. 840	40. 24024
8. 576	19. 912	30. 990	41. 4800
9. 256	20. 832	31. 1008	42. 6902
10. 300	21. 864	32. 1120	43. 8364
11. 198	22. 945	33. 1176	44. 10917

CANCELLATION.

INDUCTIVE STEPS.

1. What is the quotient of $12 \div 6$?
2. What are the factors of 12?
3. What are the factors of 6?
4. In your thought, put the factors of 12 above a line.
5. Put the factors of 6 below the line.
6. Reject the equal factors from above and below the line.
7. What factor remains?
8. Is not 2 the quotient of $12 \div 6$.
9. Will not your mental picture be like this: $\frac{2 \times \not{2} \times \not{3}}{\not{2} \times \not{3}}$?
10. Rejecting equal factors thus from dividend and divisor is called **Cancellation.**

PRINCIPLE.

Cancelling the same factor or factors in both dividend and divisor does not alter the quotient.

WRITTEN EXERCISES.

1. Divide $4 \times 5 \times 8 \times 18$ by $2 \times 3 \times 8 \times 15$.

Process.

$$\frac{\overset{2}{\cancel{4}} \times \overset{}{\cancel{5}} \times \overset{}{\cancel{8}} \times \overset{6}{\cancel{18}}}{\underset{2}{\cancel{2}} \times \underset{}{\cancel{3}} \times \underset{}{\cancel{8}} \times \underset{3}{\cancel{15}}} = 2 \times 2 = 4.$$

Explanation.

Cancelling 2 and 4 we have a resulting factor, 2; cancelling 5 and 15 we have a resulting factor, 3; cancelling 3 and 18 we have a resulting factor, 6; cancelling resulting factors 3 and 6 we have resulting factor, 2; cancelling 8 and 8 we have cancelled all the factors of the divisor, and the uncancelled factors of the dividend, 2 and 2, give us 4 for the quotient.

2. Divide $4 \times 2 \times 8 \times 24$ by $36 \times 8 \times 2$.

Process.

$$\frac{\overset{8}{\cancel{4}} \times \overset{}{\cancel{2}} \times \overset{}{\cancel{8}} \times \overset{}{\cancel{24}}}{\underset{9}{\cancel{36}} \times \underset{}{\cancel{8}} \times \underset{}{\cancel{2}}} = \frac{8}{3} = 2\tfrac{2}{3}.$$

Explanation.

We cancel as follows: 4 and 36, 2 and 2, 8 and 8, 9 and 24. 8 remains as dividend, 3 as divisor. Hence, the quotient is $2\frac{2}{3}$.

RULES.

1. Cancel out of both dividend and divisor all common factors.

2. Divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.

Query.—When no factor remains uncancelled, what is the quotient?

3. Divide, using cancellation:

1. $48 \times 3 \times 4$ by $8 \times 7 \times 5$.
2. $40 \times 10 \times 3$ by $5 \times 6 \times 2$.
3. $4 \times 5 \times 7 \times 9$ by $2 \times 2 \times 6 \times 7 \times 3$.
4. $8 \times 9 \times 12 \times 16$ by $4 \times 3 \times 5 \times 6 \times 20$.
5. $2 \times 3 \times 8 \times 12 \times 24$ by $6 \times 4 \times 36 \times 4$.
6. $18 \times 24 \times 32 \times 36$ by $9 \times 48 \times 4 \times 18$.
7. $40 \times 18 \times 13 \times 8$ by $10 \times 13 \times 16$.
8. $376 \times 14 \times 21$ by $7 \times 7 \times 16 \times 3$.
9. $5 \times 25 \times 874$ by $2 \times 437 \times 5 \times 5 \times 5$.
10. $108 \times 17 \times 9 \times 4$ by $27 \times 3 \times 16 \times 17$.
11. $15 \times 4 \times 8 \times 9$ by $30 \times 2 \times 6 \times 12$.
12. $40 \times 27 \times 32 \times 21$ by $24 \times 18 \times 16 \times 14$.
13. $30 \times 36 \times 24 \times 42$ by $45 \times 27 \times 8 \times 28$.
14. $27 \times 32 \times 45 \times 36$ by $18 \times 24 \times 9 \times 6$.
15. $6 \times 7 \times 9 \times 11$ by $2 \times 3 \times 7 \times 3 \times 21$.
16. $4 \times 14 \times 16 \times 24$ by $7 \times 8 \times 32 \times 12$.
17. $11 \times 9 \times 7 \times 15 \times 6$ by $30 \times 3 \times 21 \times 3$.
18. $55 \times 36 \times 27 \times 42$ by $12 \times 25 \times 35 \times 33$.
19. $36 \times 64 \times 25 \times 40$ by $32 \times 50 \times 18 \times 10$.
20. $56 \times 18 \times 32 \times 49$ by $16 \times 36 \times 42 \times 28$.
21. $11 \times 39 \times 14 \times 96$ by $44 \times 18 \times 26 \times 14$.
22. $2 \times 4 \times 8 \times 13 \times 7 \times 16$ by $26 \times 14 \times 8$.
23. $125 \times 60 \times 24 \times 42$ by $25 \times 120 \times 36 \times 5$.
24. $28 \times 56 \times 400$ by 112×280.
25. $56 \times 36 \times 35 \times 24$ by $40 \times 48 \times 21 \times 18$.

WRITTEN PROBLEMS.

1. How many dozen eggs, worth 15 cents a dozen, must be given for 20 pounds of sugar worth 5 cents a pound?

Process.

$$\frac{20 \times \cancel{5}}{\cancel{15}\ 3} = \frac{20}{3} = 6\tfrac{2}{3}.$$

Explanation.

The value of the sugar = 20 × 5 cents; 1 dozen eggs = 15 cents; hence, for 20 × 5 cents as many dozen eggs must be given as 15 is contained *times* in 20 × 5, which by cancellation we find to be $6\frac{2}{3}$.

2. How many books, at 15 cents apiece, may be exchanged for 12 reams of paper at 55 cents a ream?

3. How many pounds of butter, at 24 cents a pound, will pay for 45 yards of dress goods at 32 cents a yard?

4. How many pounds of butter, worth 15 cents a pound, may be bought for 25 pounds of tea at 48 cents a pound?

5. A boy bought 5 hens at 20 cents each and paid for them with apples at 10 cents a dozen. How many dozen were required?

6. How many tons of hay, at $16 per ton, must be given for 12 barrels of flour at $6 per barrel?

7. If a farmer sells 25 bushels of wheat at 60 cents a bushel and takes his pay in cloth at 40 cents a yard, how many yards of cloth does he get?

8. How many dozen eggs, at 28 cents a dozen, will pay for 84 pounds of sugar at 5 cents a pound?

9. How many bushels of potatoes, at 75 cents a bushel, must a farmer give for 36 yards of carpet worth $1.25 a yard?

10. How many bushels of oats, at 40 cents a bushel, must be given for 1600 bushels of wheat at 75 cents a bushel?

11. How many pieces of cotton cloth, each piece containing 42 yards, at 8 cents per yard, can be bought for 12 firkins of butter, each containing 56 pounds, at 20 cents a pound?

12. If 18 men can do a piece of work in 42 days, how long will it take 21 men to do the same work?

Suggestion: $\frac{\overset{2}{\cancel{42}} \times 18}{\underset{1}{\cancel{21}}}$. It will require 1 man 18 times 42 days.

13. If 1920 bricks will build a wall 15 yards long, how many bricks will be required for a similar wall 24 yards long?

14. By travelling at the rate of 20 miles a day, a person can complete a journey in 18 days. At what rate must he travel to finish it in 15 days?

15. How many pounds of coffee at 24 cents a pound would be required to pay for 3 hogsheads of sugar, each weighing 1464 pounds, and worth 4.5 cents a pound?

16. A gardener sells 75 crates of berries, 24 boxes in a crate, at 8 cents a box, and receives in return 12 rolls of matting, 40 yards in a roll. Find the price of the matting per yard?

17. Divide 285,120 by 5184, using the prime factors of each and cancelling.

NOTE.—Solve by cancellation the problems under the head of Analysis, pages 132 to 136.

FRACTIONS.

INDUCTIVE STEPS.

1. On page 115 we learned that a *Fraction* expresses one or more of the equal parts of anything. One of two equal parts is denoted by $\frac{1}{2}$, read "one-half." One of three equal parts is denoted by $\frac{1}{3}$, read "one-third." One of six equal parts is denoted by $\frac{1}{6}$, read "one-sixth."

Two of six equal parts is denoted by $\frac{2}{6}$, read "two-sixths." Six of six equal parts is denoted by $\frac{6}{6}$, read "six-sixths.'"

2. If an apple is divided into six equal parts, $\frac{6}{6}$ equals how much of the apple?

3. If an apple is divided into 10 equal parts, what fraction will express the whole of the apple?

4. If an apple is divided into 20 equal parts, what is one part called?

5. What fraction will denote the whole of the apple?

6. If 100 cents are divided equally among 5 boys, what part of the money will each boy receive?

7. What is $\frac{1}{5}$ of 100 cents?

8. Twenty cents is what part of 100 cents?

9. What is $\frac{1}{12}$ of \$144?

10. How many twelfths in the whole of any sum of money?

DEFINITIONS.

1. A **Fraction** denotes one or more of the equal parts of a unit.

2. A fraction is, moreover, the expression of a division that has not been, or cannot be, performed.

3. The dividend is called the **Numerator**, and is always placed *above* the line; the divisor is called the **Denominator**, and is always placed *below* the line.

4. The numerator and denominator are called the *terms of the fraction.*

5. The denominator shows into how many equal parts the unit or single thing has been divided.

6. The numerator shows how many equal parts form the fraction.

In $\frac{6}{7}$, *seven* is the denominator, *six* is the numerator, and 6 and 7 are the terms of the fraction. *Can the division be performed?*

7. In a **Proper Fraction** the numerator is less than the denominator.

$\frac{6}{7}$, $\frac{7}{8}$, $\frac{21}{32}$, are proper fractions.

The value of a proper fraction is less than 1. Why?

8. In an **Improper Fraction** the numerator either equals or exceeds the denominator.

$\frac{7}{7}$, $\frac{11}{10}$, $\frac{15}{13}$, are improper fractions.

The value of an improper fraction equals 1 or is greater than 1. Why?

9. A **Mixed Number** consists of an integer and a fraction.

$5\frac{6}{7}$, $20\frac{1}{3}$, $31\frac{1}{11}$, are mixed numbers.

ORAL EXERCISES.

1. Analyze the fraction $\frac{11}{12}$.

Explanation.

1. The denominator shows that the unit has been divided into 12 equal parts.
2. The numerator shows that 11 of those equal parts form the fraction.
3. The fraction is read "eleven-twelfths."
4. 11 and 12 are the *terms of the fraction.*
5. As dividend and divisor they denote $\frac{1}{12}$ of 11.

2. Analyze in like manner the following:

1. $\frac{7}{8}$, $\frac{9}{24}$, $\frac{16}{40}$, $\frac{14}{33}$, $\frac{6}{6}$, $\frac{6}{15}$, $\frac{1}{50}$, $\frac{4}{100}$, $\frac{9}{10}$, $\frac{18}{20}$, $\frac{15}{42}$, $\frac{18}{35}$, $\frac{9}{44}$, $\frac{36}{47}$.
2. $\frac{6}{11}$, $\frac{4}{20}$, $\frac{9}{27}$, $\frac{6}{5}$, $\frac{7}{10}$, $\frac{11}{31}$, $\frac{2}{80}$, $\frac{3}{8}$, $\frac{7}{11}$, $\frac{16}{21}$, $\frac{18}{35}$, $\frac{19}{40}$, $\frac{26}{43}$, $\frac{24}{60}$.
3. $\frac{11}{13}$, $\frac{17}{19}$, $\frac{8}{43}$, $\frac{8}{12}$, $\frac{9}{21}$, $\frac{2}{5}$, $\frac{3}{91}$, $\frac{7}{12}$, $\frac{15}{23}$, $\frac{15}{42}$, $\frac{19}{40}$, $\frac{24}{32}$, $\frac{18}{37}$, $\frac{35}{600}$.

3. Express by Arabic numerals:

1. Sixteen twenty-sevenths.
2. Thirteen twenty-ninths.
3. Fifteen thirty-seconds.

4. Twelve twenty-thirds.
5. Eighteen twenty-fifths.
6. Nineteen forty-seconds.
7. Eighty-one ninetieths.
8. One and 12 twentieths.
9. Two and three-ninths.
10. Nineteen and three fortieths.
11. Six sevenths.
12. Five ninths.
13. Ten fifteenths.
14. Eight twenty-thirds.
15. Three eighteenths.
16. Five twenty-fourths.
17. Fifteen thirtieths.
18. Eight twenty-sixths.
19. Seven forty-seconds.
20. Five fourths.
21. Seven elevenths.
22. Five eighths.
23. Seven ninths.
24. Eight twentieths.
25. Five thirteenths.
26. Eight twenty-firsts.
27. Eleven twentieths.
28. Five twenty-seconds.
29. Eight thirty-sixths.
30. Seven twenty-fourths.
31. Three tenths.
32. Seven fourteenths.
33. Six twentieths.
34. Eight thirty-ninths.
35. Eighteen fortieths.
36. Sixteen seventeenths.
37. Seven twenty-firsts.
38. 212 tenths.
39. 101 hundredths.
40. Nine and five-sixths.

4. What kind of numbers and fractions are:

1. $\frac{2}{3}$.
2. $\frac{5}{8}$.
3. $\frac{4}{7}$.
4. $\frac{20}{9}$.
5. $\frac{17}{18}$.
6. $\frac{8}{5}$.
7. $6\frac{2}{3}$.
8. $\frac{8}{8}$.
9. $5\frac{7}{5}$.
10. $\frac{1}{11}$.
11. $\frac{11}{12}$.
12. $\frac{9}{1}$.
13. $\frac{1728}{144}$.
14. $\frac{365}{30}$.
15. $\frac{30}{31}$.
16. $\frac{1}{100}$.
17. $\frac{2}{1000}$.
18. $\frac{3}{111}$.
19. $6\frac{7}{5}$.
20. $\frac{100}{100}$.

5. Change the following Roman numerals to Arabic, and read the fractions:

$\frac{V}{X}$. $\frac{IX}{VIII}$. $\frac{L}{C}$. $\frac{XL}{LX}$. $\frac{III}{IV}$. $\frac{VII}{VIII}$. $\frac{XIX}{XXI}$.

REDUCTION OF FRACTIONS.

INDUCTIVE STEPS.

1. Express as a fraction one fourth of a dollar.

2. Express as a fraction two fourths of a dollar.

3. Two fourths equal how many half dollars?

4. Write the equation, one half a dollar equal to two fourths.

5. Since $\frac{1}{2} = \frac{2}{4}$, how can the terms of the fraction $\frac{1}{2}$ be changed to the terms of the fraction $\frac{2}{4}$?

6. How can the terms of $\frac{2}{4}$ be changed to the terms of $\frac{1}{2}$?

7. Multiplying in like manner by 2, what does $\frac{1}{3}$ become?

8. How can $\frac{2}{6}$ be changed back to $\frac{1}{3}$?

9. How can $\frac{1}{2}$ become $\frac{3}{6}$? How can $\frac{3}{6}$ become $\frac{1}{2}$?

10. Change:

1. $\frac{1}{2}$ to 8ths.	10. $\frac{6}{12}$ to halves.
2. $\frac{1}{3}$ to 9ths.	11. $\frac{3}{15}$ to fifths.
3. $\frac{1}{4}$ to 12ths.	12. $\frac{2}{12}$ to sixths.
4. $\frac{4}{8}$ to halves.	13. $\frac{2}{3}$ to 9ths.
5. $\frac{3}{9}$ to thirds.	14. $\frac{3}{4}$ to 8ths.
6. $\frac{4}{12}$ to thirds.	15. $\frac{5}{8}$ to 16ths.
7. $\frac{1}{4}$ to 16ths.	16. $\frac{1}{2}$ to 10ths.
8. $\frac{1}{5}$ to 15ths.	17. $\frac{10}{20}$ to halves.
9. $\frac{1}{6}$ to 18th.	18. $\frac{5}{20}$ to fourths.

DEFINITIONS.

1. **Reduction** changes the form of fractions without changing their value.

2. A **Common Divisor** of two or more numbers exactly divides each of them.

$\frac{6}{9}$ becomes $\frac{2}{3}$ by dividing both terms by their common divisor 3.

3. The **Greatest Common Divisor** of two or more numbers is the greatest number that exactly divides each of them.

$\frac{6}{12}$ becomes $\frac{1}{2}$ by dividing both terms by their greatest common divisor, 6.

4. When terms have no common divisor, the fraction is said to be in its **Lowest Terms.**

$\frac{1}{2}$, $\frac{2}{3}$, $\frac{11}{15}$, are fractions in their lowest terms.

PRINCIPLE.

Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.

Reduction to Lowest Terms.

EXERCISES.

1. Reduce $\frac{36}{60}$ to its lowest terms.

Process.

$\frac{6)36}{6)60} = \frac{6}{10}$;

$\frac{2)6}{2)10} = \frac{3}{5}$.

Explanation.

1. According to the principle, we must divide both terms.

2. Dividing by 6, we obtain $\frac{6}{10}$; dividing by 2, we obtain $\frac{3}{5}$.

3. The terms 3 and 5, having no common divisor, are the lowest terms of the fraction $\frac{36}{60}$.

4. Hence, $\frac{36}{60} = \frac{3}{5}$.

How can $\frac{3}{5}$ be obtained from $\frac{36}{60}$ by one division?

What, then, is the *Greatest Common Divisor* of 36 and 60?

RULE.

1. Divide both terms of the given fraction, and also resulting terms, by any common divisor.

2. Continue thus to divide resulting terms until terms are found that have no common divisor; or,

3. Make a single division by using the Greatest Common Divisor of the given terms.

2. Reduce to their lowest terms:

1. $\frac{10}{20}$.	11. $\frac{35}{45}$.	21. $\frac{112}{119}$.	31. $\frac{357}{408}$.	41. $\frac{371}{1282}$.
2. $\frac{12}{24}$.	12. $\frac{33}{77}$.	22. $\frac{128}{132}$.	32. $\frac{819}{936}$.	42. $\frac{1344}{1536}$.
3. $\frac{15}{45}$.	13. $\frac{28}{42}$.	23. $\frac{144}{156}$.	33. $\frac{291}{388}$.	43. $\frac{999}{4995}$.
4. $\frac{18}{54}$.	14. $\frac{22}{99}$.	24. $\frac{104}{156}$.	34. $\frac{416}{918}$.	44. $\frac{2811}{4665}$.
5. $\frac{16}{48}$.	15. $\frac{36}{81}$.	25. $\frac{480}{360}$.	35. $\frac{300}{825}$.	45. $\frac{567}{783}$.
6. $\frac{30}{45}$.	16. $\frac{28}{52}$.	26. $\frac{180}{320}$.	36. $\frac{672}{784}$.	46. $\frac{480}{1248}$.
7. $\frac{20}{72}$.	17. $\frac{40}{72}$.	27. $\frac{240}{264}$.	37. $\frac{850}{935}$.	47. $\frac{576}{1125}$.
8. $\frac{21}{49}$.	18. $\frac{25}{35}$.	28. $\frac{630}{810}$.	38. $\frac{625}{1000}$.	48. $\frac{900}{3780}$.
9. $\frac{14}{56}$.	19. $\frac{20}{75}$.	29. $\frac{125}{375}$.	39. $\frac{225}{1000}$.	49. $\frac{324}{1296}$.
10. $\frac{40}{48}$.	20. $\frac{60}{85}$.	30. $\frac{450}{630}$.	40. $\frac{144}{1728}$.	50. $\frac{117}{5850}$.

Reduction to Higher Terms.

EXERCISES.

1. Reduce $\frac{5}{7}$ to 56ths.

Process.

$56 \div 7 = 8$.

$\frac{5}{7} \times \frac{8}{8} = \frac{40}{56}$.

Explanation.

1. According to the principle, we must multiply both terms.

2. $56 \div 7 = 8$, the multiplier required.

3. Multiplying both terms of $\frac{5}{7}$ by 8, we obtain $\frac{40}{56}$, a fraction in *higher terms*. Hence, $\frac{5}{7} = \frac{40}{56}$.

2. Reduce to higher terms:

1. $\frac{3}{4}$ to 12ths.
2. $\frac{5}{6}$ to 24ths.
3. $\frac{7}{8}$ to 24ths.
4. $\frac{9}{10}$ to 50ths.
5. $\frac{11}{12}$ to 48ths.
6. $\frac{5}{13}$ to 65ths.
7. $\frac{3}{20}$ to 100ths.
8. $\frac{9}{14}$ to 42ds.
9. $\frac{8}{15}$ to 45ths.
10. $\frac{6}{25}$ to 100ths.
11. $\frac{3}{4}$ to 8ths.
12. $\frac{10}{11}$ to 121sts.
13. $\frac{1}{2}$ to 125ths.
14. $\frac{\text{I}}{\text{V}}$ to 20ths.
15. $\frac{5}{16}$ to 80ths.
16. $\frac{3}{17}$ to 102ds.
17. $\frac{11}{12}$ to 144ths.
18. $\frac{9}{13}$ to 143ds.
19. $\frac{7}{15}$ to 180ths.
20. $\frac{17}{18}$ to 216ths.
21. $\frac{7}{25}$ to 300ths.
22. $\frac{9}{40}$ to 480ths.
23. $\frac{8}{31}$ to 341sts.
24. $\frac{11}{19}$ to 399ths.
25. $\frac{5}{8}$ to 64ths.
26. $\frac{12}{13}$ to 169ths.
27. $\frac{3}{4}$ to 200ths.
28. $\frac{\text{IV}}{\text{IX}}$ to 72ds.
29. $\frac{3}{11}$ to 88ths.
30. $\frac{3}{4}$ to 88ths.
31. $\frac{9}{2\frac{1}{2}}$ to 88ths.
32. $\frac{5}{24}$ to 576ths.
33. $\frac{13}{36}$ to 576ths.
34. $\frac{11}{48}$ to 576ths.
35. $\frac{17}{64}$ to 576ths.
36. $\frac{3}{14}$ to 252ds.
37. $\frac{5}{9}$ to 252ds.
38. $\frac{1}{4}$ to 252ds.
39. $\frac{11}{9}$ to 63ds.
40. $\frac{11}{21}$ to 126ths.
41. $\frac{7}{8}$ to 1000ths.
42. $\frac{\text{XIII}}{\text{XVII}}$ to 68ths.

Reduction to Common Denominator.

1. The results obtained in examples 21, 22, and 23, having the same denominator, 88, are called *Like Fractions.*

2. The results obtained in 11, 12, and 13, having different denominators, are called *Unlike Fractions.*

3. Hence, **Like Fractions** have the same denominator; **Unlike Fractions** do not have the same denominator.

4. When fractions are reduced to the same denominator they are said to have a **Common Denominator.**

5. When the common denominator is the least one obtainable, the fractions are said to have their **Least Common Denominator.**

EXERCISES.

1. Reduce $\frac{3}{4}$, $\frac{4}{3}$, $\frac{7}{12}$ to like fractions; that is, to a common denominator.

Process.

$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$

$\frac{4}{3} \times \frac{4}{4} = \frac{16}{12}$

$\frac{7}{12} = \frac{7}{12}$.

Explanation.

1. We cannot but see that by reducing $\frac{3}{4}$ and $\frac{4}{3}$ to 12ths, all the fractions will have a common denominator.

2. Both terms of $\frac{3}{4}$ multiplied by 3 gives $\frac{9}{12}$.

3. Both terms of $\frac{4}{3}$ multiplied by 4 gives $\frac{16}{12}$.

4. Hence, the results required are $\frac{9}{12}$, $\frac{16}{12}$, $\frac{7}{12}$.

2. Reduce $\frac{3}{4}$, $\frac{7}{8}$, $\frac{9}{10}$ to their least common denominator.

Process.

L. C. D. $= 40$

$40 \div 4 = 10$

$40 \div 8 = 5$

$40 \div 10 = 4$.

$\frac{3}{4} \times \frac{10}{10} = \frac{30}{40}$

$\frac{7}{8} \times \frac{5}{5} = \frac{35}{40}$

$\frac{9}{10} \times \frac{4}{4} = \frac{36}{40}$.

Explanation.

1. The least common denominator of the given fractions must be the least number that 4, 8, and 10 will exactly divide.

2. The least number that 4, 8, and 10 will exactly divide must contain the least number of factors that will produce 4, 8, and 10.

3. The L. C. D. $= \overbrace{2 \times 2 \times 2}^{8} \times \underbrace{5}_{10} = 40$.

4. $40 \div 4, 8, 10 = 10, 5, 4$, the multipliers required to reduce the given fractions to 40ths.

5. Hence, $\frac{3}{4} = \frac{30}{40}$, $\frac{7}{8} = \frac{35}{40}$, $\frac{9}{10} = \frac{36}{40}$.

Prime Factors.

$10 = 2 \times 5$

$8 = 2 \times 2 \times 2$

$4 = 2 \times 2$.

In the accompanying table of factors there are but two different factors,—2 and 5. In finding the L. C. D. of the given fractions, 2 is taken three times and 5 is taken one time, which is the greatest number of times each occurs.

Hence, the L. C. D. = the product of the different factors of the given denominators taken the greatest number of times that each occurs.

RULE.

1. Find the L. C. D. of the given fractions.

2. Reduce the fractions to the L. C. D. by the process for obtaining higher terms.

3. Reduce to a common denominator:

1. $\frac{5}{6}, \frac{7}{8}, \frac{12}{14}$.
2. $\frac{3}{4}, \frac{3}{7}, \frac{7}{8}$.
3. $\frac{3}{6}, \frac{9}{15}, \frac{18}{24}$.
4. $\frac{7}{8}, \frac{3}{4}, \frac{5}{2}$.
5. $\frac{3}{5}, \frac{3}{10}, \frac{7}{30}$.
6. $\frac{5}{8}, \frac{5}{16}, \frac{7}{48}$.
7. $\frac{3}{4}, \frac{2}{5}, \frac{3}{10}$.
8. $\frac{2}{7}, \frac{3}{4}, \frac{3}{14}$.
9. $\frac{2}{3}, \frac{5}{4}, \frac{5}{18}$.
10. $\frac{3}{11}, \frac{3}{4}, \frac{9}{22}$.
11. $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}$.
12. $\frac{2}{5}, \frac{13}{100}, \frac{19}{20}, \frac{2}{25}$.
13. $\frac{4}{5}, \frac{7}{8}, \frac{17}{20}, \frac{9}{10}$.
14. $\frac{8}{9}, \frac{11}{12}, \frac{1}{2}, \frac{1}{4}$.
15. $\frac{3}{4}, \frac{5}{13}, \frac{25}{26}, \frac{1}{8}$.
16. $\frac{4}{7}, \frac{3}{8}, \frac{2}{5}, \frac{41}{42}$.
17. $\frac{6}{10}, \frac{4}{8}, \frac{9}{12}, \frac{4}{5}$.
18. $\frac{6}{9}, \frac{12}{15}, \frac{4}{12}, \frac{9}{18}$.
19. $\frac{2}{5}, \frac{3}{7}, \frac{2}{3}, \frac{5}{9}$.
20. $\frac{7}{8}, \frac{8}{9}, \frac{10}{21}, \frac{6}{7}$.
21. $\frac{1}{6}, \frac{5}{9}, \frac{5}{12}, \frac{3}{4}$.
22. $\frac{7}{12}, \frac{3}{20}, \frac{4}{5}, \frac{7}{20}$.
23. $\frac{11}{30}, \frac{5}{28}, \frac{9}{42}, \frac{5}{21}$.
24. $\frac{3}{20}, \frac{6}{35}, \frac{9}{50}, \frac{18}{36}$.
25. $\frac{11}{48}, \frac{17}{64}, \frac{7}{12}, \frac{3}{32}$.
26. $\frac{18}{30}, \frac{15}{45}, \frac{9}{18}, \frac{18}{72}$.
27. $\frac{3}{11}, \frac{4}{7}, \frac{19}{28}, \frac{7}{33}$.
28. $\frac{7}{68}, \frac{3}{21}, \frac{5}{357}, \frac{6}{17}$.
29. $\frac{2}{25}, \frac{3}{10}, \frac{47}{50}, \frac{4}{75}$.
30. $\frac{1}{2}, \frac{4}{7}, \frac{3}{16}, \frac{2}{21}$.
31. $\frac{2}{9}, \frac{1}{21}, \frac{3}{4}, \frac{6}{1}$.
32. $\frac{9}{11}, \frac{3}{8}, \frac{4}{7}, \frac{1}{4}$.

Reduction to Mixed Numbers.

EXERCISES.

1. Reduce $\frac{293}{27}$ to a whole or a mixed number.

Process.

$$27\overline{)293}\,(10\tfrac{23}{27}$$
$$\underline{27}$$
$$23$$
$$\underline{00}$$
$$23$$

Explanation.

Division is indicated. We therefore divide 293 by 27, and obtain $10\frac{23}{27}$.

Or, we may say: "$1 = \frac{27}{27}$; therefore, in 293 twenty-sevenths there are as many 1's as 27 is contained times in 293, or $10\frac{23}{27}$."

RULE.

Perform the division indicated, and make the quotient exact.

2. Reduce to whole numbers:

1. $\frac{4}{2}$.	4. $\frac{20}{5}$.	7. $\frac{15}{3}$.	10. $\frac{40}{5}$.	13. $\frac{20}{4}$.
2. $\frac{15}{5}$.	5. $\frac{12}{3}$.	8. $\frac{18}{3}$.	11. $\frac{12}{4}$.	14. $\frac{40}{10}$.
3. $\frac{10}{2}$.	6. $\frac{30}{5}$.	9. $\frac{8}{4}$.	12. $\frac{40}{8}$.	15. $\frac{24}{4}$.

3. Reduce to mixed numbers:

1. $\frac{7}{2}$.	5. $\frac{15}{4}$.	9. $\frac{17}{3}$.	13. $\frac{13}{3}$.	17. $\frac{40}{6}$.	21. $\frac{31}{4}$.
2. $\frac{15}{2}$.	6. $\frac{16}{5}$.	10. $\frac{10}{3}$.	14. $\frac{30}{7}$.	18. $\frac{25}{8}$.	22. $\frac{14}{5}$.
3. $\frac{16}{3}$.	7. $\frac{15}{4}$.	11. $\frac{26}{3}$.	15. $\frac{21}{6}$.	19. $\frac{14}{4}$.	23. $\frac{49}{12}$.
4. $\frac{12}{5}$.	8. $\frac{16}{3}$.	12. $\frac{25}{4}$.	16. $\frac{14}{3}$.	20. $\frac{45}{8}$.	24. $\frac{45}{8}$.

4. Reduce to whole or mixed numbers:

1. $\frac{729}{27}$.	5. $\frac{3611}{44}$.	9. $\frac{2705}{1900}$.	13. $\frac{5445}{900}$.
2. $\frac{580}{48}$.	6. $\frac{5625}{50}$.	10. $\frac{1875}{30}$.	14. $\frac{2191}{56}$.
3. $\frac{841}{24}$.	7. $\frac{723}{101}$.	11. $\frac{3471}{399}$.	15. $\frac{2039}{300}$.
4. $\frac{20502}{180}$.	8. $\frac{2349}{429}$.	12. $\frac{7385}{1900}$.	16. $\frac{1899}{348}$.

Reduction of Mixed Numbers.

EXERCISES.

1. Reduce $57\frac{5}{7}$ to an improper fraction.

Process.

$57 = \frac{399}{7}$.

$\frac{399}{7} + \frac{5}{7} = \frac{404}{7}$.

Explanation.

Since $1 = \frac{7}{7}$, $57 = \frac{399}{7}$. $\frac{399}{7} + \frac{5}{7} = \frac{404}{7}$.

Or, regarding the mixed number as the result of a division, we may say: "57, the quotient, multiplied by 7, the divisor = 399; 399 + 5, the remainder = 404, the dividend. Hence, $57\frac{5}{7} = \frac{404}{7}$."

RULE.

1. **Multiply the integer by the denominator.**
2. **To the product add the numerator.**
3. **Place the sum over the denominator.**

2. Reduce to improper fractions:

1. $17\frac{3}{8}$.	5. $86\frac{5}{17}$.	9. $33\frac{9}{99}$.	13. $560\frac{19}{29}$.
2. $402\frac{11}{12}$.	6. $17\frac{11}{100}$.	10. $188\frac{2}{31}$.	14. $256\frac{31}{327}$.
3. $49\frac{15}{47}$.	7. $29\frac{53}{100}$.	11. $509\frac{7}{35}$.	15. $1307\frac{9}{75}$.
4. $62\frac{13}{18}$.	8. $93\frac{127}{246}$.	12. $312\frac{7}{23}$.	16. $1492\frac{3}{31}$.

REVIEW.

1. Define the following terms:
   1. Fraction.
   2. Numerator.
   3. Denominator.
   4. Proper Fraction.
   5. Improper Fraction.
   6. Mixed Number.
   7. Reduction.
   8. Lowest Terms.
   9. Common Divisor.
   10. Greatest Common Divisor.
   11. Like Fractions.
   12. Unlike Fractions.
   13. Common Denominator.
   14. Least Common Denominator.

2. What is the principle of reduction to higher and lower terms?

3. What is the rule of reduction to lowest terms?

4. What is the L. C. D. the product of?

5. What is the rule for reduction to the L. C. D.?

ADDITION OF FRACTIONS.

INDUCTIVE STEPS.

1. $\$\frac{1}{4} + \$\frac{2}{4}$ = how many fourths of a dollar?

2. $\frac{2}{5}$ of a day and $\frac{1}{5}$ of a day = how many fifths of a day?

3. If you spend $\frac{3}{10}$ of a dollar for pens and $\frac{7}{10}$ of a dollar for paper and ink, how many tenths and how many dollars do you spend?

4. If a girl reads $\frac{1}{4}$ of a book one week, $\frac{1}{2}$ of the book the next week, what fractional part of the book has she read?

5. Why cannot $\frac{1}{4}$ and $\frac{1}{2}$ be added directly?

6. When fractions are unlike, what change must be made upon them before they can be added?

PRINCIPLE.

If the fractions to be added are unlike, they must be reduced to like fractions.

ORAL EXERCISES.

1. Allen gave $\frac{1}{8}$ of his money for oranges and $\frac{3}{8}$ for apples. What part of his money did he spend?

2. If a book cost $\frac{3}{4}$ of a dollar and some paper $\frac{1}{5}$ of a dollar, how much did both cost?

3. Mary, having $\frac{5}{6}$ of a dollar, earned $\frac{1}{6}$ of a dollar. Had she then more or less than one dollar?

4. A boy spent $\frac{2}{3}$ of his money for a bat and $\frac{1}{6}$ of it for a ball. What part did he spend for both?

5. Three ducks cost $\frac{4}{5}$ of a dollar and two geese $\frac{10}{6}$ of a dollar. What was the entire cost?

6. $\frac{74}{30}$ of a dollar equals how many dollars?

Find the exact quotient.

7. $\frac{14}{30}$ of a dollar equals how many fifteenths?

8. I sold $\frac{3}{5}$ of an acre of land to one man, $\frac{7}{10}$ of an acre to another, and $\frac{7}{20}$ to another. How many acres did I sell?

9. A man spent $9\frac{7}{8}$ dollars for a coat and $2\frac{3}{4}$ dollars for a vest. How much money did he spend for both?

Suggestion: $9 + 2$ and $\frac{7}{8} + \frac{3}{4}$.

10. $9\frac{2}{5}$ dollars equal how many fifths of a dollar?

Analysis: $1 = \frac{5}{5}$. $9 = \frac{45}{5}$. $\frac{45}{5} + \frac{2}{5} = \frac{47}{5}$.

11. $\$2\frac{5}{8}$ = how many eighths? *Analyze.*

12. Reduce $\frac{46}{5}$ to a mixed number.

13. Reduce $\frac{27}{6}$ to a mixed number.

14. Find the sum of $9\frac{1}{5}$ and $4\frac{1}{2}$.

WRITTEN EXERCISES.

1. What is the sum of $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{7}{8}$?

Process.

1. L. C. D. $= 2 \times 2 \times 2 \times 3 \times 5 = 120$.
2. $120 \div 5, 6, 8, = 24, 20, 15$.
3. $\frac{4}{5} + \frac{5}{6} + \frac{7}{8} = \frac{96}{120} + \frac{100}{120} + \frac{105}{120} = \frac{301}{120} = 2\frac{61}{120}$.

Explanation.

1. The fractions are unlike.
2. They must be reduced to like fractions.
3. L. C. D. $= 2 \times 2 \times 2 \times 3 \times 5 = 120$.
4. 120 divided by 5, 6, and 8 give 24, 20, and 15 as multipliers of both terms.

Hence, $\frac{4}{5} = \frac{96}{120}$; $\frac{5}{6} = \frac{100}{120}$; $\frac{7}{8} = \frac{105}{120}$.

Adding, we have $\frac{301}{120} = 2\frac{61}{120}$.

$$\begin{array}{r} 96 \\ 100 \\ 105 \\ \hline 301 \end{array}$$

2. Find the sum of:

1. $\frac{5}{6}, \frac{2}{3}, \frac{7}{8}$.
2. $\frac{1}{3}, \frac{4}{7}, \frac{1}{2}$.
3. $\frac{4}{5}, \frac{7}{20}, \frac{11}{30}$.
4. $\frac{3}{7}, \frac{8}{35}, \frac{3}{5}$.
5. $\frac{1}{2}, \frac{3}{4}, \frac{17}{20}$.
6. $\frac{1}{3}, \frac{7}{10}, \frac{5}{12}$.
7. $\frac{7}{8}, \frac{3}{4}, \frac{7}{12}$.
8. $\frac{1}{2}, \frac{4}{1}, \frac{7}{8}$.
9. $\frac{1}{4}, \frac{7}{16}, \frac{5}{24}$.
10. $\frac{3}{5}, \frac{5}{6}, \frac{13}{15}$.
11. $\frac{4}{21}, \frac{7}{10}, \frac{5}{12}$.
12. $\frac{3}{5}, \frac{6}{6}, \frac{3}{10}$.
13. $4\frac{1}{3}, 5\frac{1}{2}, \frac{1}{4}$.
14. $3\frac{1}{5}, 6\frac{1}{7}, 3\frac{2}{5}$.
15. $\frac{4}{5}, \frac{17}{20}, \frac{11}{35}$.
16. $1\frac{3}{5}, 2\frac{3}{7}, 4\frac{4}{15}, 6\frac{2}{3}$.
17. $1\frac{1}{11}, 2\frac{5}{6}, 3\frac{1}{4}, \frac{15}{22}$.
18. $2\frac{11}{14}, 5\frac{6}{7}, \frac{7}{8}, 1\frac{1}{4}$.
19. $5\frac{3}{8}, \frac{8}{15}, \frac{9}{10}, 12\frac{1}{4}$.
20. $14\frac{4}{5}, 3\frac{9}{16}, 1\frac{2}{3}, \frac{19}{20}$.
21. $28\frac{5}{11}, 6\frac{5}{22}, 2\frac{12}{121}$.

3. Find the sum of $\frac{6}{18}, \frac{30}{12}, 3\frac{6}{24}$.

Process.

$\frac{6}{18} = \frac{1}{3}$,

1. $\frac{30}{12} = 2\frac{6}{12} = 2\frac{1}{2}$,

$\frac{6}{24} = \frac{1}{4}$.

2. $2 + 3 = 5$.
3. $\frac{1}{3} + \frac{1}{2} + \frac{1}{4} = \frac{4+6+3}{12} = \frac{13}{12} = 1\frac{1}{12}$.
4. $5 + 1\frac{1}{12} = 6\frac{1}{12}$.

Explanation.

1. No one of the fractions is in its lowest terms, and $\frac{30}{12}$ is an improper fraction. Reducing, we have $\frac{6}{18} = \frac{1}{3}$, $\frac{30}{12} = 2\frac{1}{2}$, $\frac{6}{24} = \frac{1}{4}$.

2. $2 + 3 = 5$, the sum of the integers.

3. $\frac{1}{3} + \frac{1}{2} + \frac{1}{4}$, reduced to a common denominator, equals $1\frac{1}{12}$.

4. 5, the sum of the whole numbers, plus $1\frac{1}{12}$, the sum of the fractions, equals $6\frac{1}{12}$.

RULE.

1. Reduce improper fractions to whole or mixed numbers.

2. Reduce all fractions in higher terms to their lowest terms.

3. Reduce the fractions to be added to a common denominator.

4. Place the sum of the numerators over the common denominator.

5. Add the sum of the integers to the sum of the fractions.

As we have already seen, whole numbers occurring should be added separately, and their sum finally added to the sum of the fractions.

4. What is the sum of:

1. $\frac{1}{2}$ and $\frac{1}{3}$?
2. $\frac{1}{2}$ and $\frac{1}{4}$?
3. $\frac{1}{3}$ and $\frac{1}{5}$?
4. $\frac{1}{3}$ and $\frac{1}{6}$?
5. $\frac{1}{3}$ and $\frac{1}{4}$?
6. $\frac{2}{3}$ and $\frac{1}{2}$?
7. $2\frac{2}{3}$ and $3\frac{3}{4}$?
8. $\frac{2}{3}$ and $\frac{2}{5}$?
9. $4\frac{3}{4}$ and $5\frac{2}{5}$?
10. $\frac{3}{4}$ and $\frac{1}{6}$?
11. $\frac{3}{5}$ and $\frac{1}{6}$?
12. $\frac{5}{6}$ and $\frac{4}{5}$?
13. $6\frac{3}{4}$ and $7\frac{3}{8}$?
14. $\frac{7}{8}$ and $\frac{2}{3}$?
15. $7\frac{2}{7}$ and $8\frac{1}{2}$?
16. $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$?
17. $\frac{1}{6}$, $\frac{1}{4}$, and $\frac{1}{5}$?
18. $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{8}$?
19. $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{3}{4}$?
20. $1\frac{2}{3}$, $2\frac{2}{5}$, and $3\frac{1}{2}$?
21. $\frac{4}{5}$, $\frac{3}{4}$, and $\frac{2}{3}$?

5. Find the sum of:

1. $\frac{13}{26}$, $\frac{51}{65}$, $\frac{24}{25}$, $\frac{1}{2}$.
2. $7\frac{1}{2}$, $5\frac{4}{12}$, $10\frac{3}{4}$.
3. $14\frac{4}{5}$, $3\frac{9}{10}$, $1\frac{2}{3}$, $\frac{20}{20}$.
4. $\frac{7}{8}$, $1\frac{7}{12}$, $10\frac{5}{6}$, 5.
5. $2\frac{1}{3}$, $\frac{3}{4}$, $5\frac{2}{7}$, $4\frac{1}{8}$.
6. $29\frac{3}{10}$, 45, $16\frac{4}{15}$.
7. $91\frac{1}{2}$, $270\frac{1}{4}$, $3\frac{2}{16}$, $\frac{1}{16}$.
8. $23\frac{8}{9}$, $65\frac{1}{3}$, $\frac{139}{6}$, $\frac{19}{2}$.
9. $\frac{25}{2}$, $\frac{20}{3}$, $\frac{33}{4}$, $\frac{95}{6}$, $\frac{3}{8}$.
10. $\frac{55}{3}$, $\frac{37}{5}$, $\frac{93}{10}$, $\frac{1}{5}$.
11. $2\frac{1}{15}$, $8\frac{1}{5}$, $27\frac{2}{3}$, $9\frac{1}{5}$.
12. $\frac{4}{6}$, $\frac{9}{15}$, $\frac{4}{40}$, $\frac{3}{20}$, $\frac{16}{25}$.
13. $63\frac{1}{10}$, $3\frac{2}{3}$, $\frac{13}{6}$, $\frac{29}{5}$.
14. $17\frac{9}{1000}$, $\frac{27}{100}$, $6\frac{3}{10}$, $\frac{15}{1000}$, $\frac{3}{100}$.
15. $48\frac{2}{3}$, 30, $63\frac{3}{4}$, $8\frac{1}{16}$, $\frac{7}{8}$.
16. $\frac{5}{13}$, $\frac{2}{39}$, $\frac{25}{78}$, $\frac{7}{156}$.
17. $\frac{1000}{24}$, $\frac{100}{8}$, $\frac{10}{3}$.
18. $\frac{99}{40}$, $7\frac{9}{25}$, $8\frac{7}{50}$.
19. $1\frac{1}{8}$, $1\frac{1}{9}$, $1\frac{1}{10}$, $1\frac{1}{11}$.
20. $\frac{21}{8}$, $\frac{22}{9}$, $\frac{23}{10}$, $\frac{24}{11}$, $\frac{25}{12}$.

WRITTEN PROBLEMS.

1. James earned \$$1\frac{4}{5}$, John \$$5\frac{3}{10}$, Asa \$$11\frac{1}{2}$, Andrew \$$8\frac{1}{4}$, and Thomas \$$7\frac{9}{10}$. How much did they all earn?

2. A man walked $26\frac{1}{3}$ miles on Monday, $13\frac{5}{7}$ miles on Tuesday, $21\frac{1}{5}$ miles on Wednesday, and $13\frac{7}{20}$ miles on Thursday. How many miles did he travel in all?

3. A man bought 4 pieces of cloth. The first contained $37\frac{5}{9}$ yards; the second, $41\frac{5}{7}$ yards; the third, $27\frac{3}{8}$ yards; the fourth, $38\frac{1}{12}$ yards. How many yards did he buy?

4. If a pound of beef costs $12\frac{5}{9}$ cents, a pound of tea \$1.21$\frac{1}{3}$, a pound of coffee $22\frac{4}{9}$ cents, what will all cost?

5. Two sheep cost $\frac{17}{2}$ of a dollar, a calf $\frac{9}{4}$ of a dollar, and a lamb $\frac{5}{4}$ of a dollar. What was the entire cost?

6. A man who had spent $9\frac{7}{8}$ dollars for a coat and $2\frac{3}{4}$ dollars for a vest had $6\frac{4}{7}$ dollars remaining. How much had he at first?

7. Find the number of pounds of butter in 4 tubs weighing $27\frac{1}{3}$ pounds, $34\frac{3}{8}$ pounds, $32\frac{4}{5}$ pounds, and $29\frac{2}{9}$ pounds.

8. I gave away \$38$\frac{3}{5}$ and had left \$5$\frac{7}{8}$. How much money had I at first?

9. A man gave his three boys \$5.00, \$7$\frac{1}{2}$, and \$8$\frac{1}{3}$ respectively, and had \$10$\frac{1}{6}$ left. How much money had he at first?

10. A bicycler rode $27\frac{3}{4}$ miles on Monday, $33\frac{1}{3}$ miles on Tuesday, $37\frac{3}{4}$ miles on Wednesday, and $42\frac{2}{5}$ miles on Thursday. How far did he ride in the four days?

11. A three-sided field has its sides $31\frac{8}{11}$, $46\frac{2}{8}$, $59\frac{12}{33}$ rods long respectively. How far is it around the field?

12. Four piles of wood contain $37\frac{3}{16}$, $41\frac{7}{8}$, $29\frac{15}{128}$, $54\frac{12}{32}$ cords respectively. How many cords are in the four piles?

13. A merchant sold $46\frac{3}{4}$ yards of cloth for $\$127\frac{7}{16}$, $64\frac{5}{8}$ yards for \$226$\frac{5}{6}$, and $76\frac{1}{2}$ yards for \$312$\frac{3}{5}$. How many yards did he sell, and how much money did he receive?

14. Find the value of $8\frac{1}{2} + \frac{3}{4} + 5\frac{1}{6} + 7\frac{8}{18}$.

SUBTRACTION OF FRACTIONS.

INDUCTIVE STEPS.

1. $\$\frac{3}{4} - \$\frac{1}{4}$ = how many fourths of a dollar?

2. $\frac{4}{5}$ of a day $- \frac{2}{5}$ of a day = how many fifths of a day?

3. If you have $\frac{7}{10}$ of a dollar and spend $\frac{3}{10}$ of a dollar, how many tenths of a dollar have you left? How many fifths?

4. Some boys have $\frac{3}{4}$ of an hour for play. When they have played $\frac{1}{3}$ of an hour, what part of an hour remains for play?

5. Why cannot $\frac{1}{3}$ be subtracted from $\frac{3}{4}$ directly?

6. When fractions are unlike, what change must be made upon them before subtraction?

PRINCIPLE.

If the fractions to be subtracted are unlike, they must be reduced to like fractions.

ORAL EXERCISES.

1. If I have $\frac{9}{10}$ of a dollar, and spend $\frac{1}{2}$ of a dollar, how much will I have left?

2. If a boy has $\frac{3}{4}$ of a dollar and gives away $\frac{3}{5}$ of a dollar, how much money has he remaining?

3. If a girl who has $\frac{4}{5}$ of a dollar gives away $\frac{3}{10}$ of a dollar, how much has she left?

4. Henry, having $\$\frac{6}{8}$, gave $\$\frac{1}{4}$ for a slate and $\$\frac{1}{8}$ for an inkstand. How much money had he left?

5. A gentleman owning a yacht sold $\frac{5}{12}$ of it to a friend. What part of it did he still own?

6. A boy earned $\$2\frac{3}{4}$ and immediately spent $\$\frac{3}{5}$. How much had he then?

Suggestion: $\$2\frac{3}{4} = \$2\frac{15}{20}$.

7. If John earns $\$\frac{7}{8}$ per day and Henry earns $\$\frac{6}{7}$, how much more does John earn than Henry?

8. If a man earns \$3 per day and his wife spends $\$2\frac{7}{8}$ thereof per day, how much is left per day?

Suggestion: $\$3 = \$2\frac{8}{8}$.

9. I spent at a store $\$1\frac{7}{12}$ and gave in payment a \$2 bill. How much did I receive in change?

10. A lady having bought a hat for $\$5\frac{1}{5}$ and gloves for $\$1\frac{3}{4}$, received how much in change out of a \$10 bill.

WRITTEN EXERCISES.

1. What is the difference between $\frac{5}{8}$ and $\frac{3}{12}$.

Process.

$\frac{3}{12} = \frac{1}{4}$;

$\frac{5}{8} - \frac{1}{4} = \frac{5}{8} - \frac{2}{8} = \frac{3}{8}$.

Explanation.

1. We first reduce $\frac{3}{12}$ to its lowest terms, $\frac{1}{4}$; and then write $\frac{5}{8} - \frac{1}{4}$.

2. We next give the fractions a common denominator by reducing $\frac{1}{4}$ to $\frac{2}{8}$. We then write $\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$.

2. From $6\frac{3}{5}$ take $2\frac{3}{7}$.

Process.

$6\frac{3}{5} = 6\frac{21}{35}$.

$2\frac{3}{7} = 2\frac{15}{35}$.

$4\frac{6}{35}$.

3. Find the value of $9\frac{2}{5} - 3\frac{4}{7}$.

Process.

$9\frac{2}{5} = 9\frac{14}{35} = 8\frac{49}{35}$.

$3\frac{4}{7} = 3\frac{20}{35} = 3\frac{20}{35}$.

$5\frac{29}{35}$.

Explanation.

1. We may subtract the integers and fractions separately.
2. Reducing the fractions to a common denominator we have $\frac{14}{35}$ and $\frac{20}{35}$.
3. $\frac{20}{35}$ cannot be subtracted from $\frac{14}{35}$.
4. 1, taken from 9, $= \frac{35}{35}$; $\frac{35}{35} + \frac{14}{35} = \frac{49}{35}$.
5. $8\frac{49}{35} - 3\frac{20}{35} = 5\frac{29}{35}$.

RULE.

1. Reduce improper fractions to whole or mixed numbers.

2. Reduce all fractions to their lowest terms.

3. Reduce the fractions to a common denominator.

4. Place the difference of the numerators over the common denominator.

5. Add the difference of the integers to the difference of the fractions.

4. Find the value of:

1. $\frac{5}{9} - \frac{1}{3}$.
2. $4\frac{1}{2} - \frac{1}{3}$.
3. $\frac{1}{7} - \frac{1}{9}$.
4. $\frac{7}{8} - \frac{2}{9}$.
5. $\frac{2}{3} - \frac{1}{2}$.
6. $\frac{2}{11} - \frac{1}{3}$.
7. $6 - 4\frac{7}{9}$.
8. $\frac{3}{8} - \frac{1}{4}$.
9. $\frac{2}{11} - \frac{1}{7}$.
10. $\frac{7}{11} - \frac{1}{5}$.
11. $1\frac{1}{3} - \frac{1}{4}$.
12. $\frac{6}{13} - \frac{5}{39}$.
13. $7\frac{5}{9} - 3\frac{2}{3}$.
14. $4 - \frac{7}{8}$.
15. $7\frac{1}{2} - \frac{1}{9}$.
16. $\frac{3}{25} - \frac{3}{70}$.
17. $\frac{6}{35} - \frac{3}{28}$.
18. $\frac{13}{21} - \frac{7}{56}$.
19. $32\frac{6}{7} - 20\frac{7}{9}$.
20. $44 - 16\frac{12}{13}$.
21. $36\frac{2}{5} - 18\frac{5}{7}$.
22. $6\frac{3}{5} - 2\frac{3}{7}$.
23. $8\frac{5}{48} - 5\frac{5}{24}$.
24. $18\frac{1}{2} - 9\frac{3}{4}$.
25. $13 - 6\frac{5}{9}$.
26. $22 - 8\frac{6}{13}$.
27. $16\frac{3}{7} - 10\frac{5}{9}$.
28. $\frac{2}{3} + \frac{1}{6} - \frac{1}{4}$.
29. $\frac{1}{7} + \frac{5}{6} - \frac{1}{2}$.
30. $3\frac{2}{7} + \frac{1}{7} - 2\frac{7}{8}$.
31. $\frac{1}{4} + \frac{2}{5} - \frac{1}{3}$.
32. $\frac{3}{5} + \frac{2}{7} - \frac{3}{10}$.
33. $\frac{2}{9} + \frac{3}{5} - \frac{7}{20}$.
34. $\frac{5}{8} - \frac{2}{5} + \frac{3}{10}$.
35. $\frac{4}{10} - \frac{1}{3} + \frac{7}{5}$.
36. $4\frac{5}{6} + 2\frac{3}{8} - 5\frac{1}{4}$.
37. $\frac{1}{7} + \frac{3}{4} + \frac{5}{8} - \frac{9}{14}$.
38. $5 - 2\frac{1}{3} + 6\frac{1}{8} - \frac{5}{12}$.
39. $5\frac{1}{9} + 3\frac{2}{3} + 2\frac{5}{27} - 10\frac{7}{8}$.
40. $10\frac{2}{9} + 4\frac{10}{27} - 6\frac{4}{9} - 1\frac{2}{8}$.
41. $10 - 4\frac{2}{3} + 12\frac{3}{8} - \frac{7}{16}$.
42. $8\frac{6}{7} + 4\frac{6}{9} - 10\frac{1}{3} - \frac{1}{21}$.

WRITTEN PROBLEMS.

NOTE.—Indicate all operations before performing them.

1. A farmer having $4\frac{1}{4}$ tons of hay bought $8\frac{3}{8}$ tons more. He then sold $7\frac{1}{2}$ tons. How much had he left?

2. After losing $\$6\frac{1}{2}$ and loaning $\$3\frac{3}{4}$, a man had $\$18\frac{3}{5}$. How much had he at first?

3. A horse cost $\$147\frac{3}{4}$ and a carriage $\$189\frac{1}{2}$. How much more did the carriage cost than the horse?

4. What number must be added to $19\frac{7}{28}$ to make $106\frac{5}{6}$?

5. A drover sold $\frac{2}{5}$ of his flock of sheep to one man, and $\frac{1}{3}$ of it to another. What part of his flock did he sell? What part of it had he left?

6. A man, having $9\frac{3}{8}$ dollars, paid $3\frac{4}{5}$ dollars for boots, and $4\frac{5}{8}$ dollars for a hat. How much had he left?

7. The sum of two numbers is $12\frac{7}{16}$. One of the numbers is $7\frac{4}{7}$. What is the other number?

8. Add together $\frac{2}{9}$, $\frac{16}{21}$, $\frac{4}{36}$, and from their sum subtract $\frac{4}{15}$.

9. $\frac{4}{13}$ of a pole is in the mud, $\frac{5}{21}$ of it is in the water, and the rest of it is in the air. What part of it is in the air?

10. From a piece of silk containing $30\frac{1}{4}$ yards, a salesman cut off $3\frac{1}{4}$ yards, $4\frac{5}{8}$ yards, and $12\frac{1}{2}$ yards. How many yards remained?

11. A man had $\frac{1}{2}$ of his sheep in one pasture, $\frac{1}{5}$ in another, and $\frac{3}{16}$ in a third, and the remainder in a fourth. What part is in a fourth?

12. How much added to $\$10\frac{3}{7}$ will make $\$15\frac{1}{3}$.

MULTIPLICATION OF FRACTIONS.

INDUCTIVE STEPS.

1. Two times $\frac{2}{5}$ equal what? Three times $\frac{2}{5}$ equal what? 5 times $\frac{1}{10}$ equal what? $\frac{5}{10}$ = what in lowest terms?

2. If 5 times $\frac{1}{10} = \frac{1}{2}$, how may the $\frac{1}{2}$ be obtained *directly?*

1. Find 6 times $\frac{2}{18}$ in lowest terms directly.
2. Find 8 times $\frac{5}{24}$ in lowest terms directly.
3. Find 10 times $\frac{9}{50}$ in lowest terms directly.

3. Is it multiplication or division that gives lower and lowest terms?

4. There are two ways of multiplying a fraction by an integer:

1. Multiplying the numerator multiplies the fraction.
2. Will you not supply the second way?

PRINCIPLES.

1. Multiplying the numerator multiplies the fraction.

2. Dividing the denominator multiplies the fraction.

To Multiply a Fraction by an Integer.

ORAL PROBLEMS.

1. If one book costs $\frac{3}{4}$ of a dollar, what will 6 books cost?

2. At $\$\frac{11}{12}$ per yard, what will 9 yards of cloth cost?

3. How much will 16 books cost at $\$\frac{3}{5}$ each?

4. James earns $\$\frac{9}{10}$ of a dollar in one day. How much will he earn in 12 days?

5. What is the cost of 15 bushels of oats at $\frac{2}{5}$ of a dollar a bushel?

6. If a man earns $\$1\frac{1}{2}$ per day, how much does he earn in 6 days?

7. At $12\frac{1}{2}$ cents a dozen, what will 6 dozen oranges cost?

8. What will 10 dozen eggs cost at $15\frac{1}{2}$ cents per dozen?

9. What is the cost of 3 yards of cloth at $\$2\frac{3}{4}$ a yard?

10. If 6 times $6\frac{5}{6}$ years equal twice my age, how old am I?

WRITTEN EXERCISES.

1. Multiply $\frac{7}{18}$ by 9.

Process.

$\frac{7}{\cancel{18}_{2}} \times \cancel{9} = \frac{7}{2} = 3\frac{1}{2}$.

Explanation.

1. Denominator is divisible by multiplier 9.
2. Dividing the denominator multiplies the fraction.
3. Cancelling 9 and 18, we have $\frac{7}{2}$.
4. Reducing $\frac{7}{2}$, we have $3\frac{1}{2}$.

2. Multiply $12\frac{3}{5}$ by 4.

Process.

$12\frac{3}{5} = \frac{63}{5}$;

$\frac{63}{5} \times 4 = \frac{252}{5} = 50\frac{2}{5}$.

Explanation.

1. Reducing $12\frac{3}{5}$, we have $\frac{63}{5}$.
2. Denominator 5 is not exactly divisible by 4.
3. Multiplying the numerator multiplies the fraction.
4. 4 times $\frac{63}{5} = \frac{252}{5} = 50\frac{2}{5}$.

RULE.

Divide the denominator or multiply the numerator of the fraction by the integer.

NOTE.—The integer and fraction of a mixed number may be multiplied separately and the results added.

3. Multiply:

1. $\frac{5}{26}$ by 13.
2. $\frac{9}{28}$ by 14.
3. $\frac{15}{18}$ by 18.
4. $\frac{13}{36}$ by 18.
5. $\frac{112}{49}$ by 7.
6. $\frac{5}{12}$ by 23.
7. $\frac{3}{4}$ by 37.
8. $\frac{4}{5}$ by 11.
9. $\frac{15}{16}$ by 25.
10. $\frac{3}{5}$ by 400.
11. $2\frac{3}{4}$ by 8.
12. $6\frac{5}{9}$ by 5.
13. $18\frac{3}{7}$ by 21.
14. $22\frac{5}{8}$ by 108.
15. $\frac{19}{27}$ by 21.
16. $\frac{105}{108}$ by 18.
17. $\frac{105}{224}$ by 32.
18. $\frac{131}{144}$ by 60.
19. $\frac{151}{156}$ by 26.
20. $\frac{228}{1728}$ by 144.
21. $\frac{231}{1492}$ by 273.

WRITTEN PROBLEMS.

1. What will 20 magazines cost at $\$\frac{3}{20}$ each?
2. At $\$\frac{5}{8}$ a yard, what will 12 yards of cashmere cost?
3. A householder bought 21 baskets of peaches, each containing $\frac{3}{4}$ bushels. How many bushels did he buy?
4. What is the value of 14 bushels of peaches at $\$\frac{7}{8}$ per bushel?
5. How much will 18 yards of silk cost at $\$4\frac{3}{8}$ per yard?
6. At $2\frac{3}{5}$ cents a pound, what will 8 pounds of chalk cost?
7. If a family consumes $5\frac{4}{7}$ barrels of flour in 1 year, how much would they consume in 9 years?
8. When flour is $\$8\frac{3}{8}$ per barrel, how much must be paid for 16 barrels?
9. Find the weight of 8 reams of paper at $14\frac{5}{7}$ pounds per ream.

10. Find the cost of 81 acres of land at $\$28\frac{7}{8}$ per acre.

11. Mr. Brown bought 21 tons of coal at $\$5\frac{1}{4}$ a ton. What was the cost of the coal?

12. What is the cost of 27 cords of wood at $\$3\frac{5}{6}$ per cord?

13. A barrel contains $31\frac{1}{2}$ gallons. How many gallons in 13 barrels?

14. 12 men make a purchase together, each paying $\$617\frac{5}{8}$. What was the cost of the purchase?

15. What will 72 yards of cloth cost, at $\$2.12\frac{1}{2}$ per yard?

16. From a chest of tea containing $45\frac{1}{4}$ pounds, $14\frac{3}{8}$ pounds are sold. How many pounds remain? What is the value of the remainder at $1.00 per pound?

To Multiply an Integer or a Fraction by a Fraction.

INDUCTIVE STEPS.

1. John had 9 marbles and lost $\frac{1}{3}$ of them. How many did he lose?

$9 \times \frac{1}{3}$ equal how many?

$9 \times \frac{1}{3} =$ how many?

$\frac{1}{3} \times 9 =$ how many?

2. Do not $\frac{1}{3}$ of 9, $\frac{1}{3} \times 9$, $9 \times \frac{1}{3}$, 9 times $\frac{1}{3}$, all mean the same thing?

3. James had 12 apples; he gave away $\frac{1}{4}$ of them. How many did he give away?

How many ways of writing the operation have you in mind?

4. What does the word "of" mean in such cases?

5. What sign will represent "of."

6. What sign stands for the word "times."

7. $\frac{3}{5}$ of 15 means what, or is the same as what?

8. Does $15 \times \frac{3}{5}$ give the same result as $\frac{3}{5} \times 15$?

9. In $\frac{3}{5} \times 15$ what numbers may be cancelled and what is the result?

10. Express $\frac{5}{6}$ of $\frac{3}{4}$ in two other ways.

Suggestion: Use cancellation and state your result.

WRITTEN EXERCISES.

1. Multiply 7 by $\frac{17}{28}$.

Process.

$$\frac{17}{28} \times \frac{7}{1} = \frac{\overset{17}{\cancel{119}}}{\underset{4}{\cancel{28}}} = \frac{17}{4} = 4\tfrac{1}{4}.$$

Or,

$$\frac{17}{\underset{4}{\cancel{28}}} \times \frac{\cancel{7}}{1} = \frac{17}{4} = 4\tfrac{1}{4}.$$

Explanation.

1. $7 \times \frac{17}{28}$ means $\frac{17}{28}$ of 7.
2. $\frac{1}{28}$ of $7 = \frac{7}{28}$.
3. $\frac{17}{28}$ of $7 = \frac{119}{28}$.
4. Cancelling 7 out of 28 and 119 we have $\frac{17}{4} = 4\frac{1}{4}$.

In practice cancel before you multiply.

2. Multiply $\frac{5}{18}$ by $\frac{9}{20}$.

Process.

$$\frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{18}}} \times \frac{\overset{1}{\cancel{9}}}{\underset{4}{\cancel{20}}} = \frac{1}{8}.$$

Explanation.

Cancelling 5 and 9 we have, as results, above the line, $1 \times 1 = 1$; below the line, $2 \times 4 = 8$. Hence, the product is $\frac{1}{8}$.

3. Find the product of $6\frac{1}{2}$ and $4\frac{1}{4}$.

Process.

$$6\tfrac{1}{2} \times 4\tfrac{1}{4} = \tfrac{13}{2} \times \tfrac{17}{4} = \tfrac{221}{8} = 27\tfrac{5}{8}.$$

Explanation.

1. Reducing the mixed number to improper fractions we have $\frac{13}{2} \times \frac{17}{4}$.
2. There are no common factors.
3. $13 \times 17 = 221$; $2 \times 4 = 8$.
4. The product is $\frac{221}{8}$ or $27\frac{5}{8}$.

GENERAL RULE.

1. Express integers and mixed numbers as fractions and indicate the multiplication.

2. Cancel all common factors.

3. Form products of the resulting numbers.

What is the principle of the second step?

4. Multiply:

1. 14 by $\frac{5}{7}$.
2. 24 by $\frac{7}{8}$.
3. 49 by $\frac{20}{7}$.
4. 29 by $\frac{1}{58}$.
5. 106 by $\frac{28}{53}$.
6. 144 by $\frac{19}{12}$.
7. 24 by $10\frac{3}{4}$.
8. 32 by $12\frac{7}{8}$.
9. 36 by $12\frac{5}{6}$.

5. Find the value of:

1. $7 \times \frac{5}{43}$.
2. $12 \times \frac{19}{60}$.
3. $11 \times \frac{3}{121}$.
4. $\frac{57}{58} \times 11$.
5. $42\frac{2}{3} \times 9$.
6. $100\frac{3}{5} \times 12$.
7. $105 \times \frac{7}{35}$.
8. $20\frac{5}{7} \times 48$.
9. $57\frac{10}{81} \times 729$.
10. $81\frac{3}{64} \times 40$.
11. $\frac{8}{9}$ of $\frac{12}{20}$.
12. $\frac{2}{3}$ of $\frac{9}{10}$.
13. $\frac{37}{40}$ of $\frac{28}{37}$.
14. $\frac{12}{35}$ of $\frac{14}{15}$.
15. $\frac{7}{8}$ of $65\frac{3}{5}$.
16. $\frac{3}{4}$ of $55\frac{5}{6}$.
17. $\frac{5}{7}$ of $3\frac{1}{5}$.
18. $\frac{15}{17}$ of $\frac{11}{27}$.
19. $\frac{11}{25}$ of $\frac{35}{44}$.
20. $\frac{21}{36}$ of $\frac{45}{49}$.
21. $2\frac{2}{3} \times \frac{3}{4}$ of $\frac{1}{5}$.
22. $3\frac{1}{5} \times \frac{7}{8} \times \frac{1}{12}$.
23. $\frac{9}{10}$ of $8\frac{1}{3} \times \frac{1}{2}$.
24. $\frac{7}{8}$ of $\frac{8}{15}$ of $\frac{25}{2}$.
25. $\frac{5}{11} \times \frac{8}{9} \times 4\frac{2}{7}$.
26. $\frac{15}{18} \times 5\frac{1}{5} \times \frac{3}{10}$.
27. $\frac{4}{25} \times 7\frac{1}{7} \times 6\frac{2}{3}$.
28. $12\frac{1}{2} \times \frac{8}{15} \times 16\frac{2}{3}$.
29. $37\frac{1}{2} \times \frac{12}{25} \times \frac{10}{11}$.
30. $8\frac{1}{2} \times \frac{5}{12} \times 1\frac{1}{17}$.

6. What is the cost of:

1. $\frac{7}{8}$ of a ton of hay at $\$12\frac{3}{4}$ per ton?
2. $32\frac{5}{8}$ yards of broadcloth at $\$4\frac{4}{5}$ per yard?
3. 8 tons of coal at $\$5\frac{3}{4}$ a ton?

4. $5\frac{1}{4}$ cords of wood at $\$6\frac{3}{4}$ a cord?
5. $\frac{4}{5}$ of an acre of land at \$100 per acre?
6. $18\frac{1}{2}$ days' work at $\$1\frac{3}{4}$ per day?
7. $50\frac{1}{4}$ pounds of sugar at $4\frac{3}{8}$ cents per pound?
8. $10\frac{3}{4}$ yards of cloth at $18\frac{3}{4}$ cents per yard?
9. 12 baseballs at $\$\frac{7}{8}$ each?
10. $87\frac{1}{2}$ pounds of paper at $3\frac{2}{3}$ cents a pound?

WRITTEN PROBLEMS.

1. How much will $4\frac{1}{2}$ yards of cloth cost at $\$1\frac{7}{8}$ a yard?

2. At $\$\frac{7}{8}$ per bushel, what cost 21 baskets of peaches, each containing $\frac{2}{3}$ of a bushel?

3. $24\frac{3}{4}$ cubic feet equal a perch of stone. How many cubic feet in $5\frac{8}{11}$ perches?

4. A man having $387\frac{1}{2}$ acres of land sold $\frac{3}{5}$ of it. How many acres did he sell?

5. Reduce to simplest form $\frac{2}{3}$ of $\frac{7}{8}$ of $9\frac{1}{3}$.

6. Mr. A. sold $\frac{5}{6}$ of $150\frac{3}{8}$ cords of wood at $\$10\frac{2}{3}$ a cord. What did he receive for it?

7. Find the cost of 28 hats at $\$2\frac{3}{4}$ each.

8. Find the weight of $8\frac{1}{2}$ reams of paper at $14\frac{5}{7}$ pounds per ream.

9. If a man saw $3\frac{2}{3}$ cords of wood in one day, how much will he saw in $\frac{5}{8}$ of a day?

10. A horse and cow were bought for \$180. The cow cost $\frac{2}{7}$ as much as the horse. Find the price of each?

Suggestion: Let $\frac{7}{7}$ = the price of the horse.

11. $5\frac{1}{2}$ yards equal 1 rod. How many yards are there in $\frac{11}{12}$ of $8\frac{1}{5}$ rods?

12. A grocer, by selling $4\frac{1}{2}$ gallons of molasses at $\$\frac{5}{8}$ a gallon, gains $\$\frac{5}{8}$. What did the $4\frac{1}{2}$ gallons cost him?

13. What is the cost of 15 boxes of starch, each containing $7\frac{1}{2}$ pounds, at $6\frac{1}{4}$ cents per pound?

DIVISION OF FRACTIONS.

INDUCTIVE STEPS.

1. $\$\frac{3}{4} \div 3 =$ what? $\frac{5}{8}$ of a ton $\div 5 =$ what? $\frac{12}{17}$ of a mile $\div 4 =$ what?

2. In dividing by 3, 5, and 4, what part of the fractions did you divide?

3. What, then, is the effect of dividing the numerator?

4. One half-dollar equals how many quarter-dollars?

Then $\$\frac{1}{2} \div 2 =$ what? $\$\frac{1}{3} \div 2 =$ what? $\frac{4}{5}$ of a ton $\div 3 =$ what? $\frac{10}{11}$ of a mile $\div 7 =$ what?

How did you obtain the results $\frac{1}{4}$, $\frac{1}{6}$, $\frac{4}{15}$, and $\frac{10}{77}$?

5. What, then, is the effect of multiplying the denominator?

PRINCIPLES.

1. **Dividing the numerator divides the fraction.**
2. **Multiplying the denominator divides the fraction.**

To Divide a Fraction by an Integer.

ORAL PROBLEMS.

1. If 5 slates can be bought for $\$\frac{10}{12}$, what is the price of each slate?

2. If 5 pounds of sugar cost $\$\frac{9}{10}$, what is the price per pound?

3. A farmer received $\$2\frac{3}{5}$ for 8 bushels of oats. How much was that per bushel?

4. I paid $\$67\frac{1}{2}$ for coal. If I bought 10 tons, how much was that per ton?

5. How many barrels of apples at \$2 per barrel can be bought for $\$8\frac{1}{4}$?

6. If 3 bushels of potatoes are sold for $\$\frac{5}{7}$, what is the price per bushel?

7. A laborer earned $\$15\frac{3}{4}$ in 9 days. How much was that per day?

8. If 4 pairs of shoes cost $16\frac{4}{5}$ dollars, how much is that per pair?

9. I paid $1\frac{1}{10}$ dollars for 5 pounds of butter. How much was that per pound?

10. If a boy can walk $16\frac{2}{3}$ miles in 5 hours, how far can he walk in one hour?

WRITTEN EXERCISES.

1. Divide $\frac{18}{31}$ by 9.

Process.

$\frac{\cancel{18}}{31} \div \cancel{9} = \frac{2}{31}$.

Explanation.

1. The numerator 18 is divisible by 9.
2. Dividing the numerator divides the fraction.
3. Cancelling 9 and 18, we have $\frac{2}{31}$.

Or, we may say: "$\frac{18}{31} \div 9 = \frac{1}{9}$ of $\frac{18}{31}$, which is $\frac{2}{31}$."

2. Divide $\frac{30}{7}$ by 9.

Process.

$\frac{30}{7} \div 9 = \frac{30}{63} = \frac{10}{21}$.

Explanation.

1. The numerator 30 is not exactly divisible by 9.
2. Multiplying the denominator divides the fraction.
3. $\frac{30}{7} \times \frac{1}{9} = \frac{30}{63} = \frac{10}{21}$.

RULE.

Divide the numerator, or multiply the denominator, of the fraction.

3. Divide:

1. $\frac{18}{15}$ by 2.
2. $\frac{18}{25}$ by 6.
3. $\frac{14}{20}$ by 5.
4. $\frac{49}{57}$ by 7.
5. $\frac{39}{51}$ by 13.
6. $\frac{18}{45}$ by 15.
7. $\frac{57}{95}$ by 19.
8. $\frac{19}{26}$ by 21.
9. $\frac{28}{41}$ by 20.
10. $\frac{25}{48}$ by 25.
11. $\frac{125}{131}$ by 5.
12. $\frac{162}{231}$ by 9.
13. $\frac{154}{161}$ by 11.
14. $\frac{168}{231}$ by 12.
15. $\frac{182}{245}$ by 13.
16. $\frac{315}{336}$ by 18.
17. $\frac{528}{351}$ by 16.
18. $\frac{308}{385}$ by 21.
19. $\frac{275}{101}$ by 341.
20. $\frac{184}{10}$ by 207.
21. $25\frac{2}{3}$ by 6.
22. $87\frac{1}{2}$ by 12.
23. $42\frac{5}{7}$ by 18.
24. $11\frac{1}{4}$ by 5.
25. $33\frac{3}{5}$ by 12.
26. $174\frac{2}{9}$ by 14.
27. $878\frac{2}{11}$ by 15.
28. $264\frac{3}{5}$ by 7.
29. $182\frac{4}{9}$ by 9.
30. $269\frac{3}{7}$ by 144.

WRITTEN PROBLEMS.

1. If 20 pencils cost $\$\frac{3}{5}$, what part of a dollar is the price of 1 pencil?

2. If 7 dozen eggs cost $\$\frac{7}{5}$, what is the cost per dozen?

3. At \$5 per yard, how much silk can be bought for $\$18\frac{3}{4}$?

4. For $\$22\frac{1}{2}$ how many yards of cloth can be bought at \$3 per yard?

5. A father divided equally among his 5 children $\$478\frac{5}{6}$. How much did each receive?

6. A., B., and C. gained in business $\$734\frac{6}{9}$. Distribute the gain equally among them.

7. I bought 25 pounds of butter for $\$5\frac{1}{2}$. How much did I pay per pound?

8. If 54 horses cost $\$4622\frac{2}{5}$, what is the cost of each?

9. A man paid $\$99\frac{21}{25}$ for 4 cows. How much was that apiece?

10. If 9 men consume $\frac{3}{4}$ of $9\frac{3}{5}$ pounds of meat in a day, how much does each man consume?

11. How many times will $16\frac{3}{4}$ gallons of cider fill a vessel that holds 3 gallons?

12. If 87 cows cost $\$3870\frac{1}{2}$, what is their average cost?

13. If 12 yards of silk are worth $\$16.46\frac{2}{3}$, what is the value per yard?

14. John can walk 21 miles in $\frac{7}{9}$ of a day. In what part of a day can he walk 1 mile?

15. How many times is 27 contained in $\frac{7}{8}$ of $\frac{1}{2}$ of $42\frac{2}{3}$?

16. If 16 tons of hay cost $\$190\frac{4}{5}$, what is the cost of 1 ton?

17. If 21 cords of wood cost $\$115\frac{1}{2}$, what is the cost of 1 cord?

18. A man paid $\$63\frac{3}{8}$ for 15 tons of coal. Find the cost per ton?

To Divide an Integer or a Fraction by a Fraction.

1. To reciprocate is *to give in return.* $3 = \frac{3}{1}$. The reciprocal of 3 or $\frac{3}{1} = \frac{1}{3}$, so called because the space below the line reciprocates or gives in return 3 for 3.

2. Since $\frac{3}{1} \times \frac{1}{3} = 1$, and $\frac{2}{3} \times \frac{3}{2} = 1$, and so on, any two quantities whose product is 1 are called *reciprocal* quantities.

3. As we have just seen, the reciprocal of 3 is $\frac{1}{3}$, and the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, or $\frac{2}{3}$ *inverted.*

4. What is the reciprocal of 4, 5, 6, 8, 10, 12, 50, 144?

5. What is the reciprocal of $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{10}$, $\frac{20}{30}$, $\frac{50}{100}$, $\frac{21}{78}$?

WRITTEN EXERCISES.

1. Divide $\frac{11}{12}$ by $\frac{5}{7}$.

Process.

$\frac{11}{12} \times \frac{7}{5} = \frac{77}{60} = 1\frac{17}{60}$.

First Explanation.

$\frac{11}{12}$ is the dividend; $\frac{5}{7}$ the divisor, and the quotient is to be found.

If 1 or $\frac{7}{7}$ be divided by $\frac{5}{7}$ the quotient is $\frac{7}{5}$. But the dividend is not 1, but $\frac{11}{12}$ of 1, and therefore the quotient required is $\frac{11}{12}$ of $\frac{7}{5}$.

$\frac{11}{12}$ of $\frac{7}{5} = \frac{11 \times 7}{12 \times 5} = \frac{77}{60} = 1\frac{17}{60}$, quotient.

Observe that in $\frac{11 \times 7}{12 \times 5}$ the divisor $\frac{5}{7}$ appears in inverted form $\frac{7}{5}$, which is called the *Reciprocal* of $\frac{5}{7}$.

The process may also be explained as follows:

Second Explanation.

According to the principle, "Multiplying the denominator divides the fraction," we have $\frac{11}{12} \div 5 = \frac{11}{12 \times 5}$. Here 5, the numerator of the fraction, is used as a divisor. But $\frac{5}{7}$ means 5 divided by 7. Therefore, the divisor, 5, is seven times as large as it should be, and the quotient obtained by dividing by 5 is only one-seventh as large as the true quotient; hence, we must multiply by 7. $\frac{11}{12 \times 5} \times 7 = \frac{11 \times 7}{12 \times 5} = \frac{77}{60} = 1\frac{17}{60}$, quotient.

Here, again, you will observe that in $\frac{11 \times 7}{12 \times 5}$, the divisor $\frac{5}{7}$ appears in inverted form $\frac{7}{5}$.

Third Explanation.

Again, both dividend and divisor may be reduced to a common denominator and the division be performed thus:

$\frac{11}{12} \div \frac{5}{7} = \frac{77}{84} \div \frac{60}{84} = \frac{77}{60} = 1\frac{17}{60}$, quotient.

From the foregoing we derive the following

RULES.

Invert the divisor and multiply. Or,

Reduce dividend and divisor to common denominator, and then divide the numerator of the dividend by the numerator of the divisor.

2. Divide:

1. $\frac{4}{7}$ by $\frac{6}{7}$.
2. $\frac{3}{8}$ by $\frac{4}{5}$.
3. $\frac{5}{9}$ by $\frac{5}{11}$.
4. $\frac{8}{9}$ by $\frac{4}{7}$.
5. $\frac{6}{11}$ by $\frac{3}{8}$.
6. $\frac{5}{12}$ by $\frac{12}{5}$.
7. $\frac{6}{11}$ by $\frac{3}{5}$.
8. $\frac{7}{16}$ by $\frac{6}{5}$.
9. $\frac{9}{12}$ by $\frac{4}{5}$.
10. $\frac{10}{11}$ by $\frac{4}{7}$.
11. $\frac{19}{31}$ by $\frac{7}{11}$.
12. $\frac{8}{29}$ by $\frac{6}{11}$.
13. $\frac{13}{25}$ by $\frac{4}{7}$.
14. $\frac{22}{38}$ by $\frac{8}{11}$.
15. $\frac{37}{50}$ by $\frac{7}{10}$.
16. $\frac{100}{144}$ by $\frac{12}{10}$.
17. $6\frac{2}{3}$ by $\frac{2}{3}$.
18. $7\frac{11}{12}$ by $3\frac{5}{6}$.
19. 25 by $8\frac{3}{7}$.
20. $14\frac{9}{10}$ by $\frac{7}{5}$.
21. $214\frac{3}{4}$ by $25\frac{11}{12}$.
22. $\frac{21}{25}$ by $4\frac{9}{10}$.
23. $6\frac{2}{9}$ by $8\frac{3}{8}$.
24. $7\frac{5}{8}$ by $11\frac{1}{11}$.

3. Divide $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{7}$ by $\frac{2}{5}$ of $\frac{7}{8}$ of $\frac{1}{6}$.

Process.

$$\frac{3}{4} \times \frac{4}{5} \times \frac{5}{7} \times \frac{5}{2} \times \frac{8}{7} \times \frac{6}{1} = \frac{360}{49} = 7\frac{17}{49}.$$

Explanation.

1. Inverting the fractions of the divisor,
2. Writing $\times$ throughout,
3. Cancelling common factors,
4. And multiplying together remaining factors,
5. We have $\frac{360}{49} = 7\frac{17}{49}$.

4. Divide $\frac{9}{10}$ of 4 by $\frac{5}{6}$ of $3\frac{1}{4}$.

Process.

$$\frac{\cancel{9}^{\,3}}{\cancel{10}_{\,5}} \times \frac{\overset{2}{\cancel{4}}}{1} \times \frac{6}{5} \times \frac{4}{\cancel{18}_{\,3}} = \frac{432}{325} = 1\frac{107}{325}.$$

Explanation.

1. 4 and $3\frac{1}{4}$ reduced becomes $\frac{4}{1}$ and $\frac{13}{4}$.
2. Inverting divisor, cancelling, and multiplying,
3. We have $\frac{432}{325} = 1\frac{107}{325}$.

5. Divide:

1. $\frac{3}{8}$ of $\frac{5}{7}$ by $\frac{3}{4}$ of $\frac{10}{21}$.
2. $\frac{4}{15}$ of $\frac{12}{16}$ by $\frac{7}{8}$ of $\frac{16}{35}$.

3. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{18}{40}$ by $\frac{5}{6}$ of $\frac{9}{15}$ of $\frac{17}{30}$.
4. $\frac{5}{16}$ of $\frac{11}{40}$ of $\frac{80}{88}$ by $\frac{8}{11}$ of $\frac{8}{13}$ of $\frac{44}{108}$.
5. $\frac{14}{15} \times \frac{9}{32}$ by $\frac{15}{17} \times \frac{3}{16}$.
6. $\frac{11}{15} \times \frac{75}{64}$ by $\frac{22}{40} \div \frac{15}{32}$.
7. $\frac{16}{21} \times \frac{105}{8}$ by $\frac{18}{25} \times \frac{42}{45}$.
8. $15\frac{2}{3}$ by $18\frac{5}{6}$.
9. $\frac{3}{5} \times \frac{5}{7} \times \frac{3}{6}$ by $\frac{2}{3} \times \frac{4}{5} \times 6$.
10. $\frac{5}{6} \times \frac{6}{11} \times \frac{3}{5} \times 22$ by $\frac{3}{7} \times \frac{5}{6} \times \frac{7}{9} \times 10$.
11. $\frac{3}{4} \times \frac{2}{7} \times \frac{7}{9} \times 3$ by $\frac{2}{5} \times \frac{6}{8} \times \frac{10}{15}$.
12. $\frac{2}{5} \times \frac{5}{7} \times \frac{7}{9} \times 9$ by $\frac{1}{2} \times \frac{3}{5} \times \frac{5}{3}$ of 2.
13. $2\frac{1}{8} \times 1\frac{1}{84}$ by $\frac{4}{17} \times \frac{8}{35}$.
14. $\frac{7}{18}$ of $3\frac{1}{12}$ by $1\frac{7}{11} \times \frac{3}{74}$.
15. $\frac{7}{8}$ of $\frac{6}{11}$ by $\frac{8}{15}$ of $\frac{7}{32}$ of $4\frac{1}{11}$.
16. $\frac{2}{3}$ of $\frac{5}{8}$ of $\frac{6}{25}$ by $\frac{2}{5}$ of $\frac{1}{2}$ of $\frac{5}{8}$.

6. Find the value of:

1. $\frac{3}{7}$ of $3\frac{1}{3} \div 6\frac{1}{3}$.
2. $\frac{4}{9} \div \frac{5}{11}$ of $\frac{6}{17}$.
3. $(\frac{4}{7} \div \frac{4}{11}) \times \frac{7}{15}$.
4. $(3\frac{6}{7} \div 3\frac{1}{4}) \times \frac{7}{4}$.
5. $\frac{3}{4}$ of $5\frac{1}{3} \div \frac{9}{11}$ of $5\frac{6}{7}$.
6. $3\frac{1}{6}$ of $3\frac{1}{3} \div \frac{4}{15}$ of $4\frac{3}{5}$.
7. $\frac{4}{9}$ of $\frac{6}{8}$ of $\frac{15}{17} \div 7$.
8. $\frac{6}{9}$ of $4\frac{1}{3}$ of $\frac{7}{21} \div 6\frac{1}{8}$.
9. $\frac{4}{11} \div (\frac{20}{7}$ of $3\frac{1}{5}$ of $2\frac{6}{7})$.
10. $\frac{4}{11} \div (\frac{3}{5} \times 3\frac{1}{5} \times 2\frac{6}{7})$.
11. $(\frac{5}{8} \div \frac{3}{7}) \times 1\frac{7}{20}$.
12. $(\frac{4}{7}$ of $\frac{2}{3}$ of $5\frac{1}{4}) \div (\frac{15}{16}$ of $48)$.
13. $(5\frac{1}{3} \div 18\frac{2}{5}) \times (11\frac{11}{12} \div 12\frac{1}{10})$.
14. $(\frac{7}{9}$ of $2\frac{1}{7} \div 2\frac{1}{9}) \times \frac{2}{3} \div (\frac{1}{2}$ of $\frac{1}{6})$.
15. $(\frac{2}{3}$ of $\frac{3}{8}$ of $2\frac{1}{3}) \div (\frac{1}{2}$ of $3\frac{2}{5})$.
16. $(7\frac{5}{8} \div 4\frac{6}{9}) \div \frac{2}{3}$ of $\frac{9}{5}$ of $(5\frac{3}{4} \div 8\frac{5}{7})$.

17. $(31\frac{1}{3} \div 8) \times (6\frac{3}{4} \div 2\frac{1}{12}) \div 4\frac{23}{100}$.

18. $2\frac{1}{4} \times \frac{4}{7} \times (2\frac{1}{2} \div \frac{3}{8})$ of $\frac{14}{15}$.

19. $(3\frac{1}{4} \div 4\frac{1}{3}) \times \frac{12}{7} \times (\frac{7}{9} \div 3)$.

20. $(3\frac{3}{8}$ of $1\frac{7}{22}) \div (\frac{2}{5}$ of $6\frac{2}{3})$.

WRITTEN PROBLEMS.

1. I paid \$12 for baseballs, at $\frac{3}{4}$ of a dollar each. How many did I buy?

2. At $\frac{1}{8}$ of a dollar a pound, how much butter can be bought for $\frac{11}{16}$ of a dollar?

3. If a man pays $\$1\frac{1}{3}$ per day for his board, for how long a time will \$25 pay for his board?

4. How many times can a jar holding $\frac{1}{4}$ of $\frac{2}{3}$ of a gallon be filled from another jar containing $\frac{3}{4}$ of $3\frac{1}{3}$ gallon?

5. At $\$3\frac{2}{5}$ a cord, how many cords of wood can be bought for \$40?

6. At $\$\frac{3}{8}$ a pound, how many pounds of butter will \$110 buy?

7. At $\$6\frac{1}{3}$ a bushel, how many bushels of clover-seed can be bought for $\$40\frac{3}{4}$?

8. How many books, at $\$3\frac{1}{3}$ per volume, can be purchased for $\$31\frac{1}{4}$?

9. If $\frac{5}{16}$ of an acre of land is worth $\$23\frac{3}{4}$, what is one acre worth?

10. How many sheep must I sell at $\$3\frac{1}{4}$ a head to obtain \$169?

11. If a yard of silk costs $\$2\frac{4}{5}$, how many yards can be bought for $\$18\frac{9}{10}$?

12. If a man earns $\$1\frac{2}{3}$ a day, in how many days will he earn $\$12\frac{1}{2}$?

13. I paid $\$38\frac{1}{2}$ for $6\frac{7}{8}$ yards of cloth. What was the price per yard?

14. How many pounds of butter, at $32\frac{1}{2}$ cents a pound, must be given for $37\frac{1}{2}$ pounds of sugar, at 6 cents a pound?

15. By what must $2\frac{2}{3} \div 3\frac{1}{5}$ be multiplied to give the product 1?

16. If a man travel $28\frac{4}{7}$ miles in one day, how many days will it take him to travel $177\frac{2}{3}$ miles?

17. $453\frac{1}{3} \div 5\frac{5}{6} \times 11\frac{1}{3} =$ what?

COMPLEX FRACTIONS.

1. Reduce $\frac{\frac{4}{5}}{\frac{7}{9}}$ to a simple fraction.

Process.

$$\frac{4}{5} \div \frac{7}{9} = \frac{4}{5} \times \frac{9}{7} = \frac{36}{35} = 1\frac{1}{35}.$$

Explanation.

1. $\frac{4}{5}$ is dividend and $\frac{7}{9}$ divisor.
2. We invert the divisor and multiply.
3. The product is $\frac{36}{35}$, or $1\frac{1}{35}$.

2. Reduce $\frac{2\frac{1}{4}}{6\frac{1}{5}}$ to a simple fraction.

Process.

$$2\frac{1}{4} \div 6\frac{1}{5} = \frac{9}{4} \div \frac{31}{5} = \frac{9}{4} \times \frac{5}{31} = \frac{45}{124}.$$

Explanation.

1. $2\frac{1}{4}$ is dividend, $6\frac{1}{5}$ divisor.
2. Reducing the mixed numbers, inverting the divisor, and multiplying, we have $\frac{45}{124}$.

3. Find the value of:

1. $\frac{\frac{4}{7}}{\frac{2}{3}}$.
2. $\frac{9}{\frac{1}{4}}$.
3. $\frac{5\frac{3}{7}}{\frac{4}{5}}$.
4. $\frac{\frac{2}{3}}{\frac{3}{5}}$.
5. $\frac{\frac{3}{5}}{\frac{6}{7}}$.
6. $\frac{6\frac{2}{3}}{8\frac{1}{2}}$.
7. $\frac{11\frac{1}{2}}{\frac{1}{7}}$.
8. $\frac{\frac{5}{11}}{4\frac{3}{5}}$.
9. $\frac{4\frac{1}{2}}{5\frac{1}{4}}$.
10. $\frac{2\frac{5}{7}}{4\frac{1}{3}}$.
11. $\frac{\frac{2}{3}\text{ of }\frac{1}{4}}{\frac{1}{2}}$.
12. $\frac{\frac{2}{3}\text{ of }\frac{3}{5}}{\frac{1}{3}\text{ of }4\frac{1}{2}}$.
13. $\frac{1\frac{1}{3}\times 6}{\frac{2}{3}\text{ of }6}$.
14. $\frac{\frac{2}{3}\text{ of }\frac{1}{3}\text{ of }\frac{11}{16}}{\frac{1}{6}\text{ of }\frac{5}{15}\text{ of }\frac{17}{30}}$.
15. $\frac{\frac{9}{16}\text{ of }\frac{22}{27}}{\frac{21}{25}\text{ of }\frac{17}{18}}$.
16. $\frac{3\frac{1}{3}\times 1\frac{7}{8}}{\frac{9}{11}\text{ of }8\frac{1}{4}}$.
17. $\frac{\frac{21}{25}}{\frac{14}{15}}\times\frac{\frac{44}{75}}{\frac{77}{84}}$.
18. $\frac{\frac{17}{35}\div\frac{34}{35}}{1\frac{13}{105}\div 1\frac{11}{14}}$.

FRACTIONAL RELATION.

INDUCTIVE STEPS.

1. \$5 is $\frac{1}{2}$ of how many dollars? \$10 is $\frac{1}{3}$ of how many dollars? \$20 is $\frac{1}{5}$ of how many dollars? \$20 is $\frac{2}{5}$ of how many dollars? \$20 is $\frac{4}{5}$ of how many dollars?

2. Of what number is 20 four-fifths? 8 is $\frac{2}{3}$ of what number? 8 is $\frac{8}{9}$ of what number?

3. Since 8 is $\frac{2}{3}$ of 12, $\frac{2}{3}$ expresses the relation of 8 to 12.

4. Since 8 is $\frac{8}{9}$ of 9, $\frac{8}{9}$ expresses the relation of 8 to 9.

5. 8 is $\frac{2}{3}$ of what number? Since 8 is $\frac{2}{3}$ of 12, and since $\frac{8}{1}\times\frac{3}{2}=12$, how may the required number be found?

PRINCIPLE.

The given number is the dividend.
The fraction of relation is the divisor.
The required number is the quotient.

EXERCISES.

1. 84 is $\frac{6}{7}$ of what number?

Process.

$$\frac{84}{1} \div \frac{6}{7} = \frac{\overset{14}{\cancel{84}}}{1} \times \frac{7}{\cancel{6}} = 98.$$

Explanation.

1. 84 is the dividend.
2. $\frac{6}{7}$ is the divisor.
3. The quotient is required.

4. $\frac{\cancel{14}}{1} \times \frac{7}{\cancel{6}} = \frac{14}{1} \times 7 = 98.$

Or, we may say, "$\frac{6}{7}$ of the number $= 84$, $\frac{1}{7} = 14$, $\frac{7}{7} = 98$."

RULE.

Multiply by the fraction of relation inverted.

2. To apply the rule:
   1. 18 is $\frac{2}{3}$ of what number?
      a. The fraction of relation inverted is $\frac{3}{2}$.
      b. $\frac{\cancel{18}}{1} \times \frac{3}{\cancel{2}} = \frac{54}{2} = 27.$
   2. 60 is $\frac{3}{7}$ of what number?
   3. 125 is $\frac{5}{6}$ of what number?
   4. 216 is $\frac{36}{37}$ of what number?
3. To apply analysis:
   1. 47 is $\frac{2}{3}$ of what number?
      $\frac{2}{3}$ of number $= 44$; $\frac{1}{3}$ of number $= 22$; $\frac{3}{3}$ of number $= 66$.
   2. 45 is $\frac{3}{5}$ of what number?
   3. $\frac{3}{4}$ of $\frac{2}{3}$ is $\frac{1}{6}$ of what number?
   4. $1\frac{3}{5}$ is $\frac{4}{3}$ of $\frac{3}{5}$ of what number?

WRITTEN PROBLEMS.

1. If \$64 is $\frac{8}{7}$ of my money, how much money have I?

2. Sold a watch for \$43$\frac{3}{4}$, which was $\frac{7}{8}$ of its cost. What did it cost?

3. What is the price of land, per acre, when $\frac{3}{10}$ of an acre costs \$44.25.

4. If \$425 is $\frac{17}{54}$ of my salary, what is my salary?

5. Sold my farm for \$3360, which was $\frac{6}{7}$ of its value. Find its value.

6. A house was sold for $\frac{7}{9}$ of its cost. If the selling price was \$2100, what was the cost?

7. A freight train ran 15 miles per hour, which was $\frac{3}{8}$ as fast as an express train. What was the rate of the express train?

8. If \$6000 is $\frac{3}{8}$ of the value of my farm, what is $\frac{3}{4}$ of its value?

9. If \$96 is the cost of $\frac{3}{8}$ of an acre, what will one acre cost?

10. If $\frac{5}{6}$ of a box of oranges cost \$5.50, what will one box cost? 8 boxes?

11. $\frac{1}{3}$ of a foundry is worth \$540$\frac{3}{4}$. What is the foundry worth?

12. If $\frac{4}{5}$ of an acre of land cost \$80, what will 1 acre cost? What will $\frac{7}{15}$ cost?

13. $\frac{6}{7}$ of a barrel of flour costs \$4.20. Find the cost of a barrel.

14. $\frac{6}{11}$ of $5\frac{3}{4}$ is $\frac{6}{13}$ of what number?

15. A man sold $\frac{4}{5}$ of his share of stock for \$5120. What was his share worth? If he owned $\frac{1}{10}$ of the stock, what was the stock worth?

16. If $\frac{2}{15}$ of a task can be done in $\frac{5}{9}$ of a day, in what time can the whole task be performed?

17. If $\frac{3}{5}$ of a box of pens costs 25 cents, what do 18 boxes cost?

18. Mr. Brown sold a horse for $\frac{5}{6}$ of its cost, and received \$75. What was the cost of the horse?

19. A farmer sold $\frac{6}{7}$ of his sheep, keeping 20. How many sheep had he at first? How many did he sell?

20. A man bequeathed to his wife \$36,000, which was $\frac{8}{17}$ of his estate. The remainder was divided equally among his 4 children. What did each child receive?

21. If $\frac{2}{3}$ of the gain equals $\frac{5}{18}$ of the cost, what part of the cost does the whole gain equal?

ORAL REVIEW.

1. Find the value of $\frac{1}{2} + \frac{3}{4} + \frac{3}{4}$.
2. Find the sum of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$.
3. In $\$6\frac{3}{4}$ how many fourths of a dollar are there?
4. Reduce to improper fractions: $7\frac{2}{5}$, $6\frac{4}{7}$, $5\frac{3}{8}$, $7\frac{4}{9}$, $8\frac{7}{10}$.
5. Reduce to integers or mixed numbers: $\frac{14}{7}$, $\frac{10}{5}$, $\frac{16}{8}$, $\frac{65}{13}$, $\frac{100}{25}$, $\frac{1000}{125}$, $\frac{1728}{144}$.
6. What is the sum of $6\frac{1}{4}$, $4\frac{3}{4}$, $7\frac{1}{2}$, $4\frac{1}{2}$?
7. What is the value of $\$42\frac{1}{4} - \$39\frac{1}{2}$?
8. Find the difference between 20 acres and $6\frac{7}{8}$ acres.
9. If a man spends $\frac{6}{7}$ of his money, what fraction of it has he left?
10. I spent $\$16\frac{3}{4}$ and had $\$11\frac{3}{4}$ left. How much money had I at first?
11. If \$36 is $\frac{4}{5}$ of what an article cost, what did the article cost?
12. A man sold a watch for \$70, which cost him only $\frac{6}{7}$ of that sum. How much did he gain by the sale?
13. What will 3 pairs of shoes cost at $\$3\frac{3}{4}$ a pair?
14. If a horseless carriage runs $60\frac{5}{7}$ miles in 6 hours, at what rate per hour does it run?
15. A farmer bought a horse and a cow. The cow cost him \$30. $\frac{5}{6}$ of this sum is $\frac{1}{4}$ of what he paid for the horse. What did he pay for the horse?

16. If $\frac{5}{6}$ expresses the relation of $\frac{7}{8}$ to some other number, what is that number?

17. $66\frac{2}{3}$ is $\frac{2}{3}$ of what number?

18. $\frac{3}{5}$ of a number + $\frac{1}{3}$ of $\frac{3}{5}$ of the number = 40. What is the number?

WRITTEN REVIEW.

1. Find the sum of $1\frac{9}{10}$, $7\frac{8}{33}$, $\frac{2}{3}$.

2. A man having $\$27\frac{3}{4}$ received $\$16\frac{9}{10}$ for work, and paid out $\$18\frac{1}{8}$. How much had he then?

3. A boy has $\$3\frac{7}{10}$ to buy a dog worth $\$5\frac{1}{8}$. How much more money must he get?

4. How much will 6 men earn in $6\frac{1}{4}$ days at $\$3\frac{1}{3}$ each per day?

5. A person spending $\frac{1}{3}$, $\frac{2}{5}$, and $\frac{1}{8}$ of his money had $119 left. How much had he at first?

6. How much will $2\frac{3}{4}$ dozen eggs cost at $12\frac{1}{2}$ cents a dozen?

7. A farmer sold $22\frac{1}{2}$ pounds of butter at $\frac{3}{10}$ of a dollar a pound, and took in exchange cloth at $\frac{3}{4}$ of a dollar a yard. How much cloth did he get?

8. How many books at $66\frac{2}{3}$ cents each can be bought for $36.

9. What is the value of 54 loads of wheat, each containing 25 bushels, at $\$1\frac{1}{8}$ per bushel?

10. After spending $\frac{2}{5}$ of his money, a man had $36 remaining. How much had he at first?

11. A farmer buys a horse for $140, and sells it at an advance of $\frac{3}{20}$ of the cost. What is the selling price?

12. Find the value of $\frac{\frac{3}{4} \text{ of } \frac{1}{2} \text{ of } 2\frac{1}{2}}{\frac{1}{2} \text{ of } 3\frac{1}{5}}$.

13. The sum of two numbers is $12\frac{7}{16}$. One of the numbers is $7\frac{1}{4}$. What is the other number?

14. A grocer, having $\frac{7}{8}$ of a barrel of sugar, sold $\frac{3}{4}$ of it for $4\frac{5}{6}$ dollars. What was the value of the barrel at the same rate?

15. A man owns $87\frac{5}{16}$ acres of land, his wife owns $42\frac{5}{8}$ acres, and his son $29\frac{5}{8}$ acres. How many acres do they own together?

16. Of a certain farm, $\frac{1}{6}$ is pasture, $\frac{5}{8}$ is under cultivation, and the remainder is woodland. If the woodland is 50 acres, how many acres in the whole farm?

17. A wind blowing $28\frac{4}{5}$ miles an hour blows how far in $10\frac{1}{2}$ hours?

18. If A. can do a piece of work in 4 days, how much of it can he do in 1 day? If B. can do a piece of work in 5 days, how much can he do in 1 day? How much can A. and B. do together in 1 day?

19. If A. and B. can do $\frac{1}{4} + \frac{1}{5}$ of the work in one day, in what time can they together do the whole work?

20. Two men together earned \$870. If one earned $\frac{3}{4}$ as much as the other, how much did each earn?

21. What number divided by $\frac{9}{13}$ equals $6\frac{6}{17}$? What principle is involved?

22. At $\$2\frac{1}{5}$ per barrel, how many barrels of apples can be bought for \$55?

23. A dealer in farming implements paid $\$149\frac{1}{3}$ for 8 ploughs. For how much apiece must he sell them to gain $\$8\frac{2}{3}$ on each plough?

24. A man bought 14 tons of hay at $\$12\frac{1}{2}$ a ton, and sold it at $\$16\frac{5}{8}$ a ton. How many dollars did he gain? How many dollars did he gain per ton?

25. How many bushels of oats at $\frac{3}{5}$ of a dollar a bushel will pay for $\frac{3}{8}$ of a barrel of flour at $\$4\frac{3}{4}$ a barrel?

26. When wheat sells at $\$1\frac{1}{8}$ per bushel, how many bushels can be bought for \$198?

27. From $\frac{2}{5}$ of $\frac{3}{5}$ of $\frac{5}{8}$ of $3\frac{1}{3}$ subtract $\frac{1}{2}$ of $\frac{2}{3}$, and reduce to lowest terms.

28. If $\frac{3}{4}$ of a pound of tea costs $\$\frac{1}{2}$, how many pounds can be bought for $\$7\frac{1}{2}$?

29. Find the difference between $3\frac{1}{3} \times 6\frac{2}{5}$ and $7\frac{1}{3} \div 1\frac{3}{5}$.

30. A man sold $\frac{1}{2}$ and $\frac{1}{3}$ of his farm, and had $26\frac{2}{3}$ acres left. How many acres had he at first?

31. What will 75 men earn in $18\frac{3}{4}$ days, if each earns $2\frac{2}{15}$ dollars each day?

32. If it takes 11 men $45\frac{2}{3}$ days to do a piece of work, how many days will it take 1 man to do the same work?

33. Reduce to its lowest terms $\frac{6}{6}\frac{5}{7}\frac{3}{9}\frac{7}{5}$.

34. Reduce $\frac{2\frac{1}{2} \div 4\frac{2}{5}}{8\frac{1}{5} - 3\frac{1}{3}}$ to a simple fraction.

35. If 3 be added to both terms of the fraction $\frac{5}{6}$, will the value be increased or diminished?

36. $\frac{5}{6}$ of 72 is $\frac{3}{4}$ of what number?

37. Find the sum of $2\frac{7}{15}$, $4\frac{5}{9}$, $3\frac{1}{6}$.

38. Find the value of $728 - \frac{3}{4} - \frac{1}{7} - \frac{3}{5} - \frac{1}{8}$.

39. How long is a post of which 5 feet is above water, $\frac{3}{5}$ is in the water, and $\frac{2}{9}$ in the mud?

40. Reduce to its simplest form $\frac{\frac{2}{3} \times \frac{4}{5} - \frac{2}{5} \times \frac{1}{4}}{4\frac{1}{4} - 1\frac{2}{3} + 2\frac{1}{3}}$.

41. Reduce $\frac{4}{5}\frac{0}{2}\frac{0}{3}\frac{4}{6}$ to its lowest terms.

42. Find the smallest number that will exactly contain 15, 18, 21, 24, and 30.

43. Find the prime factors of 1226, 1938, and 2346, and also the G. C. D. of these numbers.

44. Find the cost of 8 rolls of carpet, $42\frac{1}{2}$ yards in a roll, at $91\frac{2}{3}$ cents per yard.

45. A horse and cow were bought for \$180; the cow cost $\frac{2}{7}$ as much as the horse. Find the price of each.

46. If $\frac{5}{6}$ of $\frac{7}{9}$ of a ship cost \$70,000, what is $\frac{8}{11}$ of it worth?

47. Divide $\frac{4}{6\frac{1}{7}} - \frac{1}{7}$ by $\frac{8}{11}$.

48. A. and B. can do a piece of work in 10 days. A. can do $\frac{1}{2}$ as much as B. In what time can each do the work?

49. From $\frac{8}{7}$ of $\frac{7}{15}$ take $\frac{3}{40}$ of $1\frac{1}{3}$.

50. Find the value of MDCCCXCIX $\div \frac{\text{IX.}}{\text{V.}}$.

51. Find the value of:

1. $(16\frac{2}{5} + 14\frac{1}{4}) \times \frac{7}{9}$.
2. $(\frac{3}{5}$ of $\frac{6}{11} \div 15) \times (15 \div \frac{3}{4}$ of $\frac{6}{7})$.
3. $(\frac{28}{81} \times \frac{45}{56}) \div (\frac{44}{54} \times \frac{42}{32})$.
4. $(\frac{1}{3} + \frac{8}{15}) \times \frac{3}{11} + (\frac{4}{15} + \frac{5}{6}) \times 3$.
5. $(\frac{2}{5}$ of $8\frac{1}{3} - 3 \times \frac{3}{10}) \times (\frac{7}{8} \div \frac{1}{4} + 5\frac{7}{8})$.
6. $(\frac{8}{15} - \frac{3}{10} - \frac{1}{40} + \frac{4}{11}) \div 281$.
7. $(7\frac{7}{10} \times 4\frac{6}{11} + 4\frac{1}{2} - 3\frac{4}{5} + 12\frac{3}{35}) \times 70$.
8. $\frac{3\frac{1}{4}$ of $\frac{8}{7}}{7\frac{3}{7}$ of $\frac{4}{11}} \div \frac{2\frac{3}{8}}{10} \times 17\frac{1}{3}$.
9. $(7\frac{1}{3} - 3\frac{3}{4} + 2) \div (4\frac{1}{2} + 3\frac{5}{8} - 2\frac{1}{3})$.
10. $\left(\frac{1\frac{1}{4}}{4\frac{1}{2}} \div \frac{2\frac{1}{3}}{2\frac{1}{4}}\right) \times \frac{4}{5}$ of $\frac{1}{2}$.

52. What is the principle of:

1. Addition of fractions? What is the rule?
2. Subtraction of fractions? What is the rule?

53. What are the principles of:

1. Multiplication of fractions? What is the rule?
2. Division of fractions? What is the rule?
3. Fractional relation? What is the rule?

DECIMAL FRACTIONS.

INDUCTIVE STEPS.

1. 1 ten = 10.

1. How often does 10 occur as a factor of 100? Then $100 = 10 \times 10$.
2. How often does 10 occur as a factor of 1000? Then 1000 = what?
3. How often does 10 occur as a factor of 10,000? Then 10,000 = what?

2. **Decimal** [Latin, *decem*, ten] means *consisting of tens.* $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, $\frac{1}{10000}$, etc., are called **Decimal Fractions** or **Decimals**, on account of their decimal denominators.

3. Decimals are fractions having 1 with ciphers annexed for their denominators.

1. $\frac{1}{10}$ of $\frac{1}{10}$ = what?
2. $\frac{1}{10}$ of $\frac{1}{100}$ = what?
3. $\frac{1}{10}$ of 1000 = what?

4. What law of increase and decrease governs both Decimals and Integers?

5. Since 10 of any order of decimals equal 1 of the next higher order, the denominator of a decimal may be indicated by the position of the numerator.

6. The numerator is always preceded by a mark [.] called the **Decimal Point.**

1. $\frac{1}{10} = .1$. First place.
2. $\frac{1}{100} = .01$. Second place.
3. $\frac{1}{1000} = .001$. Third place.
4. $\frac{1}{10000} = .0001$. Fourth place.
5. One *and* one-tenth is written 1.1.
6. One *and* one-hundredth is written 1.01.

7. The decimal point, being thus used to separate the integer and decimal of a number, is called the **Separatrix.**

1. $.5 = \frac{5}{10}$, read "five tenths."
2. $.05 = \frac{5}{100}$, read "five hundredths."
3. $.005 = \frac{5}{1000}$, read "five one-thousandths."
4. $.0005 = \frac{5}{10000}$, read "five ten-thousandths."
5. $.00005 = \frac{5}{100000}$, read "five hundred-thousandths."
6. $.000005 = \frac{5}{1000000}$, read "five millionths."

Fix in your mind that 6 decimal places express *millionths.*

Numeration Table.

Hundreds.	Tens.	Units.		Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.
3	4	5	.	6	3	2	1	7	4
Integers.				Decimals.					

The orders below millionths are: Ten-millionths, hundred-millionths, billionths, ten-billionths, hundred billionths, etc.

In the table, what place is held by:

1. Tenths?
2. Millionths?
3. Hundredths?
4. Thousandths?
5. Ten-thousandths?
6. Millionths?
7. Hundred-thousandths?
8. Ten-thousandths?
9. Hundredths?
10. Tenths.

In the table, what is the decimal name of the:

1. First place?
2. Sixth place?
3. Second place?
4. Fifth place?
5. Third place?
6. Fourth place?
7. Sixth place?
8. Fifth place?
9. Fourth place?
10. Third place?
11. Second place?
12. First place?

5.6 is read "Five *and* 6-tenths."

45.63 is read "Forty-five *and* 63-hundredths."

345.632 is read "Three hundred forty-five *and* 632-thousandths."

The decimal point is read "and."

In reading a decimal, only the decimal name of the last figure is given.

EXERCISES.

1. Read 53.467.

1. The integer is read "Fifty-three."
2. The point is read "and."
3. 467 is read "Four hundred sixty-seven thousandths."
4. 53.467 is read "Fifty-three *and* four hundred sixty-seven thousandths."

NOTE.—When there is no integer the point is not read.

RULE.

1. **Read the decimal as you read an integer.**
2. **Close with the decimal name of the right-hand figure.**

2. Read:

1. .38.
2. .58.
3. .487.
4. .056.
5. .0579.
6. 3.2.
7. 4.03.
8. 5.004.
9. 6.0005.
10. 7.00006.
11. 300.45.
12. 126.567.
13. 75.890.
14. 87.0781.
15. 999.00089.

16. .5346.	21. 81.000007.	26. 1000.321467.
17. .7935.	22. 92.123456.	27. 5000.000078.
18. .80465.	23. 100.789012.	28. 6789.000005.
19. .915766.	24. 246.345678.	29. 1234.123456.
20. .0268778.	25. 757.009102.	30. 5678.901234.

3. Write 169 thousandths as a decimal.

1. Writing the given number as an integer, we have 169.
2. Prefixing the decimal point, we have .169.

4. Write as a decimal 36 ten-thousandths.

1. Writing the number as an integer, we have 36.
2. Ten-thousandths occupy the fourth place.
3. Prefixing two ciphers, we have 0036.
4. Prefixing the decimal point to that result, we have .0036.

RULE.

1. Disregarding the decimal name, write the given number as an integer.

2. When necessary, prefix ciphers to give the last digit the decimal name required.

3. To the result prefix the decimal point.

5. Write as decimals the following:

1. Eight tenths. Seven tenths. Six tenths. One tenth.
2. Twenty-five hundredths. Thirty-two hundredths.
3. Twenty-seven thousandths. Three hundred three thousandths.
4. Eight ten-thousandths. Ninety-five hundred-thousandths.
5. Eight *and* three hundred seventeen millionths.
6. Twelve *and* seven hundred thirty-three thousandths.

7. Fifty *and* one hundred seven ten-thousandths.
8. Forty-eight *and* fifty-five hundred-thousandths.
9. Eighty-four *and* nine millionths.
10. 537 hundred-thousandths. 47 millionths.
11. 840 ten-thousandths. 435 thousandths.
12. 507 millionths. 480 ten-thousandths.
13. 46 hundred-thousandths. 420 thousandths.
14. 326 ten-millionths. 25 billionths.
15. 27 hundredths. 11 thousandths. 6 ten-thousandths.
16. 3 millionths. 4 ten-thousandths. 5 ten-millionths.
17. Forty-five *and* two hundred seventy-five thousandths.
18. Six *and* twenty-five ten-thousandths.
19. 21,875 hundred-thousandths.
20. One *and* one thousand one-millionths.
21. Two hundred thirty-one millionths.
22. 2051 *and* 42 hundredths.
23. 3 *and* 14 hundred 16 ten-thousandths.

6. Change to the decimal form:

1. $\frac{8}{10}$.
2. $\frac{75}{100}$.
3. $\frac{87}{1000}$.
4. $\frac{265}{1000}$.
5. $\frac{21}{10000}$.
6. $\frac{35}{1000}$.
7. $\frac{27}{100}$.
8. $\frac{441}{10000}$.
9. $\frac{425}{1000000}$.
10. $\frac{4}{100000}$.
11. $326\frac{495}{100000}$.
12. $394\frac{42}{10000}$.
13. $265\frac{5}{1000000}$.
14. $341\frac{6}{10}$.
15. $527\frac{86}{1000}$.
16. $431\frac{425}{100000}$.
17. $1899\frac{365}{1000000}$.
18. $247\frac{8}{10000}$.
19. $1900\frac{376}{100000}$.
20. $1899\frac{281}{1000000}$.
21. $\frac{128}{1000}$.
22. $7\frac{9}{1000}$.
23. $\frac{3906}{10000}$.
24. $127\frac{12}{100}$.
25. $\frac{572}{100000}$.
26. $84\frac{308}{10000}$.
27. $48\frac{175}{10000}$.
28. $218\frac{3046}{1000000}$.
29. $1\frac{10106}{100000}$.
30. $70\frac{63}{100000}$.

REDUCTION OF DECIMALS.

Unlike to Like Decimals.

INDUCTIVE STEPS.

1. $\frac{7}{10}$ of a dollar = how many cents?
   1. $\frac{70}{100}$ of a dollar = how many cents?
   2. Does $\$\frac{7}{10} = \$\frac{70}{100}$?
2. What is the difference, then, between .7 and .70?
3. What effect, then, has annexing a cipher to a decimal?

PRINCIPLE.

Annexing a cipher to a decimal does not alter its value.

EXERCISES.

1. Reduce .5, .47, and .046 to like fractions.

Process.

.5 = .500
.47 = .470
.046 = .046

Explanation.

1. Thousandths is the lowest denomination.
2. We must reduce .5 and .47 to thousandths.
3. Annexing ciphers, .5 = .500, and .47 = .470.

Principle: Annexing ciphers to a decimal does not alter its value.

RULE.

Give all the decimals the same number of figures by annexing ciphers.

2. Reduce the following to like decimal fractions:
   1. .69, .034, .0576.
   2. .4, .0536, .00576.
   3. .06, .005, .005742.
   4. .004, .053, .00456.
   5. 4.6, .573, 43.0568.
   6. 3.9, 5.24, .34056.
   7. 9.2, 1.146, 86.1136.
   8. .72, 31.57, .52405.
   9. .004, 4.05, 4.0057.
   10. 9.2, 24, .057246.

3. Reduce the following to like decimals:

1. .08, .75, .006, 3.079.
2. .000135, 1.4067, 13.025.
3. 63, 71.455, 315.7005, 6.15.
4. .409, 3.61, 75, .10055, 19.6.

A Decimal to a Common Fraction.

EXERCISES.

1. Reduce .86 to its equivalent common fraction.

Process.

$.86 = \frac{86}{100} = \frac{43}{50}$.

Explanation.

1. Two decimal places express hundredths.
2. Therefore, $.86 = \frac{86}{100}$.
3. $\frac{86}{100}$ reduced $= \frac{43}{50}$.

RULE.

1. **Omit the decimal point.**
2. **Write the denominator.**
3. **Reduce the fraction to its lowest terms.**

2. Reduce to common fractions:

1. .36.	11. .0625.	21. .0075.	31. .058.
2. .75.	12. .0375.	22. .112.	32. .0725.
3. .45.	13. .0750.	23. .405.	33. .0065.
4. .50.	14. .0500.	24. .0032.	34. .0562.
5. .95.	15. .0875.	25. .0048.	35. .5064.
6. .500.	16. .00500.	26. .3525.	36. .58$\frac{1}{3}$.
7. .375.	17. .05625.	27. .0108.	37. .83$\frac{1}{3}$.
8. .625.	18. .47043.	28. .0002.	38. .008$\frac{2}{3}$.
9. .875.	19. .270496.	29. .0006.	39. .583$\frac{1}{3}$.
10. .125.	20. .000047.	30. .0014.	40. .003$\frac{3}{4}$.

A Common Fraction to a Decimal.

INDUCTIVE STEPS.

1. 1 = how many tenths?

2 = how many tenths?

$\frac{1}{5}$ of 2, or $\frac{2}{5}$, = how many tenths?

2. If we annex 0 to 2, making the fraction $\frac{20}{5}$, and point off one decimal place in the quotient, what will be the result?

$\frac{4}{50}$ = how many hundredths?

3. If we annex 00 to 4, making the fraction $\frac{400}{50}$, and point off two decimal places in the quotient, will not the result be the same, .08?

PRINCIPLE.

A decimal place must be cut off in the quotient for every cipher annexed to the numerator.

EXERCISES.

1. Reduce $\frac{3}{8}$ to an equivalent decimal.

Process. $\frac{3.000}{8} = .375.$

Explanation.

1. To render the division exact we annex three decimal ciphers.
2. Dividing 3000 thousandths by 8, we obtain 375 thousandths.
3. Pointing off three decimal places, we have .375.

2. Reduce $\frac{6}{7}$ to a decimal.

Process. $\frac{6.00}{7} = .85\frac{5}{7}.$

Explanation.

1. 7 will not exactly divide a number ending in 0.
2. We must, however, annex ciphers.
3. 600 hundredths divided by 7 = $85\frac{5}{7}$ hundredths.
4. Pointing off two decimal places, we have .$85\frac{5}{7}$.

In many cases the common fraction may be omitted as unimportant.

3. Reduce to decimals:

1. $\frac{5}{16}$.	8. $\frac{19}{21}$.	15. $\frac{1}{41}$.	22. $\frac{75}{128}$.	29. $\frac{3}{125}$.
2. $\frac{23}{41}$.	9. $\frac{51}{161}$.	16. $\frac{1}{81}$.	23. $\frac{51}{256}$.	30. $\frac{4}{375}$.
3. $\frac{5}{8}$.	10. $\frac{19}{64}$.	17. $\frac{15}{18}$.	24. $\frac{21}{61}$.	31. $12\frac{5}{16}$.
4. $\frac{7}{8}$.	11. $\frac{3}{75}$.	18. $\frac{1}{80}$.	25. $\frac{5}{9}$.	32. $17\frac{3}{16}$.
5. $\frac{13}{125}$.	12. $\frac{8}{25}$.	19. $\frac{13}{25}$.	26. $\frac{10}{33}$.	33. $14\frac{71}{81}$.
6. $\frac{37}{32}$.	13. $\frac{5}{148}$.	20. $\frac{17}{21}$.	27. $\frac{12}{625}$.	34. $515\frac{11}{75}$.
7. $\frac{5}{16}$.	14. $\frac{7}{211}$.	21. $\frac{27}{78}$.	28. $\frac{321}{32}$.	35. $231\frac{4}{125}$.

ADDITION OF DECIMALS.

Since decimals and integers belong alike to the decimal system, the process of adding decimals does not essentially differ from that of adding integers, which requires the numbers and orders to be added to be of like name.

PRINCIPLE.

If the decimals to be added are unlike, they must be reduced to like decimals.

EXERCISES.

1. Find the sum of 2.47, 4.364, .0564.

Process.

2.47	= 2.4700
4.364	= 4.3640
.0564	= .0564
6.8904	6.8904

Explanation.

1. The given fractions are unlike.

2. They must be reduced to like fractions.

3. The lowest given denomination is ten-thousandths.

4. Annexing ciphers, .47 becomes .4700; .364 becomes .3640.

5. Writing like orders in the same column and adding, we have 6.8904.

The process shows that the decimal points of the given numbers, and that of their sum, stand in the same vertical line, and that in practice the ciphers required by reduction may be omitted.

2. Find the sum of:

1. 4.36, .537, 49.52.
2. 2.7, 43.54, .0546.
3. 46, 3.486, 2.057.
4. 3.2, 4.394, 57.3.
5. .4679, 33.10, .536.
6. 16.39, 25.46, 32.84.
7. 46.38, .2375, 29.54.
8. 3.054, 42.307, .0006.
9. 47.5, 2.736, 42.439.
10. 495.3, 2.604, 5.3976.
11. 4.670, 379, 42.574.
12. 46.74, 37.9, 357.60.
13. .3295, 32.95, 329.5.
14. 37.54, 27.986, 38.45.

3. Find the sum of:

1. 30.062, 57.6203, 5620.07.
2. 105.7, 5.0027, 29.9, 947.13.
3. 400.07, 27.4, 987.09, 4.019, 470.9.
4. 4.07, 39.0625, 900.07, 36.065, 219.107.
5. 23.873, .5625, 678.9, 19,719, 56.81.
6. 625.25, 20.029, 3075.33, 927.8, 729.006.
7. 7.29, 39.3039, 809.14, 90.075, 71.5.
8. 301.5, 7.512, 6140.11, 114.3, 9.763.
9. 27.725, .6833, 9080.09, 78.006, 214.72.
10. 151.39, 19.058, 1900.07, 6.705, 80.8.
11. 23.04, 8.6796, .0005, 7.00019, $8\frac{999}{1000}$, $\frac{3}{25}$.

PROBLEMS.

1. What is the sum of four and 47 hundredths, five and 758 thousandths, twenty-five and 475 thousandths?

2. What is the sum of 897 and 9 ten-thousandths, 17 millionths, 18 thousandths, 98 ten-millionths, 167 hundred-thousandths, and 195 ten-millionths.

3. A merchant's sales were as follows: On Monday,

$470.45; on Tuesday, $307.29; on Wednesday, $584.40; on Thursday, $579.48; on Friday, $225.36; on Saturday, $617.21. Find the total amount of his sales.

4. $52\frac{7}{10}$, 240, $34\frac{8}{1000}$, $12\frac{1557}{10000}$, $3\frac{87}{10000}$.

5. A farmer sold at different times the following quantities of hay: 3.75 tons, 14.165 tons, 375.16247 tons, 54.8125 tons, 18.5 tons, 21.75 tons, and 25 tons. How many tons did he sell?

SUBTRACTION OF DECIMALS.

PRINCIPLE.

If the decimals to be subtracted are unlike, they must be reduced to like decimals.

EXERCISES.

1. From 35.34 subtract 9.6735.

Process.		Explanation.
35.34	= 35.3400	1. The given decimals are unlike.
9.6735	= 9.6735	2. They must be reduced to like decimals.
25.6665	25.6665	3. The lowest denomination is 10,000ths.

4. Annexing ciphers, .34 becomes .3400.

5. Writing like orders in the same column, and subtracting, we have 25.6665.

NOTE.—Be careful to place a point in the remainder directly under the points in the given numbers.

2. What is the difference between:

1. 38.46 and 26.53?
2. 26.53 and 14.575?
3. 47.49 and 32.576?
4. 94.43 and 77.486?
5. 52.97 and 33.40?
6. 47.53 and 24.355?
7. 38.29 and 7.5467?
8. 42.3 and 22.654?
9. .37 and 5.683?
10. 7.386 and 4.3956?
11. 5.94 and .5947?
12. 4.39 and .0547?
13. .76 and .076?
14. .0294 and .001426?
15. .108 and .0456?
16. 400.07 and 27.4?
17. 900.07 and 36.065?
18. 301.5 and 7.512?
19. 6140.11 and 114.3?
20. 27.725 and .6833?

3. Find the value of:

1. 3.46 — .075 + 4.34 — 2.3466.
2. 4.683 + 3.47 — .526 — 3.7243.
3. 6.24 + .430 — 5.275 — .00056.
4. 5.7 + 3.4607 — 2.005 — 4.4.
5. 3.8004 — 1.00005 + 4.8 — 5.0506.
6. 400.07 — 27.4 + 987.09 — 4.019 + 470.9.
7. 7.29 + 39.3039 + 809.14 — 90.075 — 71.5.
8. 301.5 — 7.512 + 6140.11 — 114.3 — 9.763.
9. 27.725 — .6833 + 9080.09 — 78.006 — 214.72.
10. 151.39 + 19.058 + 1900.07 — 6.705 — 80.8.

PROBLEMS.

1. From a piece of cloth containing 67.35 yards, $24\frac{5}{8}$ yards were cut. How many yards remained?

2. A metre is 39.3704 inches. A seconds-pendulum is 39.1392 inches in length. Find the difference of length.

3. Two men walk, respectively, 26.7 miles and 22.94 miles per day. How much farther does the first walk than the second?

4. Find the value of \$385.75 — \$197.89.

5. Find the value of $84 \times 1.13 - (66 - 1.2 \times 2.4) + 100 \times (4 \times .018 + .189)$.

6. A clerk has a yearly salary of \$1000. He pays \$312 for board, \$157.50 for clothing, and \$372.25 for all other expenses. How much does he save in a year?

7. A man who owed \$699.60, paid \$164.87. How much did he still owe?

8. Find the sum and difference of .79864 and .801.

9. A merchant owned 64.803 acres of land. He bought 10.7045 acres, and sold all but 16.455 acres. How many acres did he sell?

10. A lady's dress cost \$13¾, her bonnet \$5½, her shoes \$2⅜, her fan \$⅞. Twenty-five dollars were presented in payment. How much change was received?

MULTIPLICATION OF DECIMALS.

INDUCTIVE STEPS.

1. $\frac{1}{10} \times \frac{1}{10} =$ what? $.1 \times .1 =$ what?

2. Both factors have how many decimal places? Their product has how many decimal places?

3. $\frac{1}{10} \times \frac{1}{100} =$ what? $.1 \times .01 =$ what?

4. Both factors have how many decimal places? Their product has how many decimal places?

PRINCIPLE.

The product of two decimals contains as many decimal places as both the decimals.

EXERCISES.

1. What is the product of .426 and .34?

Process.	Explanation.
.426	1. $426 \times 34 = 14484$.
.34	2. Both factors have $3 + 2$, or 5 decimal places.
1704	3. We must point off 5 decimal places in the product?
1278	4. $.426 \times .34 = .14484$. [Principle.]
.14484	Or, we may say, "$.426 = \frac{426}{1000}$ and $.34 = \frac{34}{100}$; $\frac{426}{1000} \times \frac{34}{100} = \frac{14484}{100000} = .14484$."

2. Multiply .125 by .06.

Process.	Explanation.
.125	1. $125 \times 6 = 750$.
.06	2. 5 decimal places are required.
.00750	3. Prefixing ciphers we have .00750.

3. Multiply:

1. .43 by .48.
2. .53 by 3.5.
3. .67 by 39.
4. .83 by .406.
5. .93 by .057.
6. 1.027 by .425.
7. .936 by 3.74.
8. .534 by 4.7.
9. 32.8 by .045.
10. 4.34 by .0067.
11. .270 by 4.053.
12. 35.3 by 4.86.
13. 27.9 by .036.
14. .923 by .0045.
15. 4.57 by .00537.
16. .3254 by .0053.
17. .2704 by .00476.
18. 1.905 by .0345.
19. 20.27 by .0057.
20. 34.08 by .00365.
21. 34.05 by 4.2706.
22. 406.03 by 2.5.
23. 7.09 by .0304.
24. 30.701 by .575.
25. 937.06 by .65.
26. 9.704 by 40.7.
27. 19.07 by $.16\frac{2}{3}$.
28. 11095 by 2.9.
29. 3.097 by .075.
30. .0035 by .0005.

PROBLEMS.

1. What will 300 bushels of wheat cost at $1.25 per bushel?

2. A merchant sold 12.35 yards of silk at $3.15 per yard. How much did he receive for it?

3. Find the cost of 20.5 tons of hay at $12.375 a ton.

4. Bought 14.75 yards of gingham at 14 cents a yard. What was the cost of the piece?

5. What is the cost of 976 yards of cloth at $1.37½ per yard?

6. Find the cost of 140 sacks of guano, each sack containing 162½ pounds, at $17¾ a ton.

7. What is the value of 1648 bushels of wheat at $.62½ per bushel?

8. What cost 250 yards of carpet at $1.60 per yard?

9. What cost 64 barrels of apples at $2.50 per barrel?

10. What cost 34.8 yards of cloth at 87½ cents per yard?

11. What cost 24 yards of silk at $1.16⅔ per yard?

12. What will 840 bushels of oats cost at 25 cents a bushel?

13. What will 26 loads of lime cost, each containing 15½ bushels, at 22½ cents a bushel?

14. What are 17.6 acres + 23.25 acres + 42.625 acres worth at $40 per acre?

15. What are 45 dozen eggs worth at $.12½ per dozen?

16. 231 cubic inches = one gallon; 31.5 gallons = 1 barrel. How many cubic inches in a barrel?

17. A farmer bought .75 bushels of grass seed at $5 a bushel. Find the cost.

18. A man owning .4236 of a yacht sold .3 of his share. What part had he left?

DIVISION OF DECIMALS.

INDUCTIVE STEPS.

1. $\frac{3}{10} \times \frac{3}{10}$ = what? .3 × .4 = what? What are .3 and .4 called in relation to their product? Factor × factor = what? .3 × .04 = what product? How many decimal places are in the product?

2. The product must always be given as many decimal places as there are in what?

3. The dividend is the product of what two factors?

4. The dividend contains as many decimal places as both the —— and the ——.

5. If the dividend has 6 decimal places and the divisor has 4 decimal places, how many decimal places must the quotient have?

PRINCIPLE.

The quotient contains as many decimal places as the number of decimal places in the dividend exceeds the number in the divisor.

EXERCISES.

1. Divide .08128 by .32.

Process.

```
2     5     3
.32 ).08128 ( .254
      64
      ---
      172
      160
      ---
       128
       128
       ---
```

Explanation.

1. 8128 ÷ 32 = 254.

2. .08128 has 5 decimal places; .32 has 2 decimal places.

3. The quotient must have 5 — 2, or 3 decimal places.

4. Hence, the quotient required is .254.

Proof.

.254 × .32 = .08128.

2. Divide 17.28 by .00144.

Process.

```
5     5      0
.00144)17.28000(12000
       144
       288
       288
       000
```

Explanation.

1. The divisor .00144 has 5 decimal places; the dividend cannot have a less number.

2. Annexing three ciphers to 17.28, we have 17.28000.

3. $5 - 5 = 0$; the quotient, therefore, can have no decimal place.

4. Dividing, we have for quotient the integer 12,000.

3. Divide 7.3 by 3650.

Process.

```
0     3     3
3650)7.300(.002
     7.300
```

Explanation.

1. Annexing two ciphers to 7.3, we have 7.300.

2. 3650 has *no* decimal place; 7.300 has 3 decimal places.

3. The quotient must have 3 — 0, or 3 decimal places.

4. Hence, the quotient required is .002, obtained by prefixing two ciphers.

RULE.

1. Divide without regard to the decimal point.

2. Should the dividend lack figures, annex ciphers.

3. After dividing, give the quotient as many decimal places as the number of decimal places in the dividend exceeds those in the divisor.

4. Should the quotient lack figures, prefix ciphers.

4. Divide:

1. 2.176 by .34.
2. .07245 by .23.
3. 16.5 by .25.
4. .0864 by .24.
5. 3.024 by .07.
6. 2.1824 by .034.
7. 405.15 by .111.
8. 45.625 by .125.
9. .0125 by 2.5.
10. 58.794 by 12.3.

11. .0043 by .230.
12. .065 by .50.
13. 7 by 350.
14. 1.2 by 3.60.
15. 75 by .025.
16. 1.075 by .43.
17. 1.5652 by .043.
18. 3.024 by .07.
19. 739.44 by .009.
20. 185.175 by .015.
21. 10.24 by 320.
22. 600 by .625.
23. 6.256 by .375.
24. .03876 by .19.
25. 40 by 640.
26. 7.6 by .0304.
27. .18312 by .056.
28. .12126 by .235.
29. .0169 by .013.
30. 80.010 by .009.
31. 16.1262 by 3.06.
32. 1.3621 by .514.
33. .016074 by .047.
34. 1.25 by .015.
35. 65 by .0039.
36. .0402 by 150.
37. 3647 by .125.
38. 72 by .064.
39. 16.02 by .045.
40. 34.4088 by 1.62.
41. .291624 by 2.32.
42. 30,000 by .000003.
43. 102.102 by 102.
44. 1.1502 by .0027.
45. .0342568 by .006523.
46. .987650 by .0000125.

The Decimal Point as Multiplier and Divisor.

INDUCTIVE STEPS.

1. .001 = what? 0.01 = what? 00.1 = what? What has been done with the decimal point? How does .01 compare in value with .001? How does .1 compare in value with .01?

2. Moving the decimal point one place to the right has what effect upon a number? Moving the point two places has what effect? Three places? Four places?

3. .1 = what? .01 = what? .001 = what? What has here been done with the decimal point? How does .01

compare in value with .1? How does .001 compare in value with .01? How does .001 compare in value with .1?

4. Moving a decimal point to the left has what effect upon a number? Moving the point two places has what effect? Three places? Four places? Five places?

PRINCIPLES.

1. Every removal of a decimal point one place to the right multiplies the number by 10.

2. Every removal of a decimal point one place to the left divides the number by 10.

EXERCISES.

1. Multiply 7.943 by 100.

Process.	Explanation.
794.3	1. There are two ciphers in the multiplier. 2. We therefore move the point two places towards the right and have 794.3.

2. Multiply:

1. 39.63 by 10.
2. 49.306 by 100.
6. .95436 by 10,000.
7. .8 by 10; by 100; by 1000.

NOTE.—If there are not enough places, annex ciphers.

3. 3.946 by 1000.
4. .495 by 100.
5. 6.387 by 10.
8. .2000 by 1,000,000.
9. .00013 by 100,000.
10. .3041 by 10,000.

3. Multiplication by a decimal has the effect of making the product less than the multiplicand; hence, to multiply by .1, .01, or .001, etc., we move the point toward the left.

4. Multiply:

1. 39.63 by .1.
2. 49.306 by .01.
3. 3.946 by .001.
4. .495 by .01.

WRITTEN EXERCISES.

1. Divide 436.58 by 100.

Process. **Explanation.**

4.3658. 1. There are two ciphers in the multiplier.

2. Therefore, we move the point two places toward the left, and have 4.3658.

2. Divide:

1. 403.6 by 100.
2. 3756 by 10.
3. 470.6 by 1000.
4. 4.825 by 100.
5. 38.62 by 1000.
6. 53.95 by 10,000.
7. .8 by 10; by 100; by 1000.
8. .00013 by 100,000.
9. 2000 by 1,000,000.
10. .3041 by 100,000.

Division by a decimal has the effect of making the quotient larger than the dividend; hence, to divide by .1, .01, or .001, etc., we move the point toward the right.

3. Divide:

1. 403.6 by .01.
   $403.6 \div .01 = 40360.$
2. 37.56 by .1.
3. 470.6 by .001.
4. 756.9 by .0001.
5. 4.825 by .001.

PROBLEMS.

1. I bought a farm containing 125 acres for $6843.75 What was the price per acre?

2. At $3 a yard, how many yards of cloth can be bought for $546?

3. If 4.7 acres of land produce 131.6 bushels of wheat, what is the average crop per acre?

4. How many dozen eggs, at \$.12½ a dozen, can be bought for \$12?

5. How many yards of velvet, at \$4 a yard, can be bought for \$23?

6. How many pounds of tea can be bought for \$6.75 at 75 cents a pound?

7. At \$12.375 a ton, how many tons of hay can be bought for \$2326.50?

8. How many tons of freight at \$2.12½ per ton can be transported for \$107.07?

9. At \$10.45 per barrel, how many barrels of flour can be bought for \$1055.45?

10. At \$.31¼ a bushel, how many bushels of potatoes can be bought for \$9?

11. How many dress patterns of 12.50 yards each can be cut from 4 pieces of French muslin containing 25 yards each?

12. Find a man's daily wages when he was paid \$27.70 for 22 days' work?

13. Paid \$40.50 for a pile of wood, at the rate of \$3.37½ a cord. How many cords were in the pile?

14. How many oranges, at 8⅓ cents each, will \$1.50 buy?

15. Find the price of each:

1. If 250 bushels of corn cost \$125.00.
2. If 70 pounds of sugar cost \$2.80.
3. If 288 bushels of wheat cost \$259.20.
4. If 200 acres of land cost \$4267½.
5. If 18 turkeys cost \$15.75.

6. If 792 pounds of rice cost $39.60.
7. If 2000 pounds of butter cost $350.
8. If 2500 pounds of beef cost $156.25.
9. If 1234 pineapples cost $98.72.
10. If $4\frac{3}{4}$ tons of coal cost $25.41\frac{1}{4}$.

16. Find the value of:

1. $2.24 \times 6 \div 28$.
2. $\$30.10 \div 10 \times 1000$.
3. $8.2 \times 9.3 - (45\frac{3}{8} \div 12.5)$.
4. $(25 \times .5 \times 12 + 20) \div 100$.
5. $3.71 + 2.64 \div 160 + 7.55 \times .07 + .071 \times 25$.
6. $(15 - 10 \times .3) \times 6.192 \div (7 \times 5.4 - 35.048)$.
7. $(4.625 + 1.146) - (1.2 + 3.571)$.
8. $1.5 \times .08 \times .5$.
9. $94.5 \div 250 + 16 \div (4.5 \div .225) + 87.25 \div (1.6 - .35)$.
10. $(\overline{.48 \div 800} \times 10{,}000 + \overline{6.4 \div .08}) \div .125$.

UNITED STATES MONEY.

United States Money, as we have already seen, is expressed in the decimal system. Its denominations and their relation to one another are as follows:

Table.

10 mills	make 1 cent (c.).
10 cents	make 1 dime (d.).
10 dimes	make 1 dollar ($).
10 dollars	make 1 eagle (E.).

The dollar is the unit.

$5\frac{1}{2}$ is written decimally $5.50, or $\$5\frac{50}{100}$.

$5 and 5 cents is written decimally $5.05, or $\$5\frac{5}{100}$.

$5 and 5 mills is written decimally $5.005.

1. Read the following:

1. $25.85.	6. $22.36.	11. $670.086.
2. $26.965.	7. $210.210.	12. $3056.002.
3. $35.355.	8. $256.006.	13. $6789.012.
4. $255.236.	9. $300.003.	14. 3456.789.
5. $36.05.	10. $505.505.	15. $1798.365.

2. Write the following in figures:
   1. Seven dollars, twenty-five cents.
   2. Ten dollars, forty cents, five mills.
   3. Forty-five dollars, fifty-four cents.
   4. Sixty-five dollars, eight cents.
   5. Seven eagles, seven dollars, seven cents.
   6. Two hundred dollars, six and a half cents.
   7. Ninety dollars, eight dimes, $8\frac{1}{2}$ cents.
   8. One thousand fifty-six dollars, $94\frac{1}{2}$ cents.
   9. Eight thousand seventy-nine dollars.
   10. One dollar, one cent, one mill.

ORAL EXERCISES.

1. $1 equals how many cents? $\$\frac{1}{2}$? $\$\frac{1}{4}$? $\$\frac{1}{10}$?
2. $4 equal how many cents?
3. $\frac{1}{2}$ cent equals how many mills? $\frac{1}{5}$ cent?
4. $\$\frac{1}{8}$ equals how many cents? $\$\frac{5}{8}$? $\$\frac{7}{8}$?
5. $1 equals how many mills?
6. 20 mills equal how many cents? 40 mills?
7. 30 cents equal how many dimes? 50 cents?
8. 30 dimes equal how many dollars? 60 dimes?
9. 200 cents equal how many dollars? 500 cents?
10. $20 equal how many eagles? $40?
11. 10 cents is what part of a dollar? 20 cents? 50 cents?

12. 25 cents is what part of a dollar? 30 cents? 75 cents?

The following rules are obvious:

1. **To change:**
   (1) **Cents to mills, multiply by 10.**
   (2) **Dollars to cents, multiply by 100.**
   (3) **Dollars to mills, multiply by 1000.**
2. **To change:**
   (1) **Mills to cents, divide by 10.**
   (2) **Cents to dollars, divide by 100.**
   (3) **Mills to dollars, divide by 1000.**

NOTE.—The proper placing of the decimal point in a given number will effect the change required.

13. Change:

1. 12,345 mills to dollars.
2. Six cents and six mills to dollars.
3. 57 cents to mills.
4. 43 mills to cents.
5. 46 cents to mills.
6. 57 mills to cents.
7. 47 dollars to cents.
8. 35 dollars to cents.
9. 296 dollars to cents.
10. 326 dollars to cents.
11. 4600 cents to dollars.
12. 900 cents to dollars.
13. 836 cents to dollars.
14. 2548 cents to dollars.
15. 26 dollars to mills.
16. 9 dollars to mills.
17. 2700 mills to dollars.
18. 2956 mills to dollars.
19. 3548 mills to dollars.
20. $70.30 to cents.
21. $27.35 to cents.
22. $29.03 to cents.
23. $56.38 to cents.
24. $7.866 to mills.
25. 5795 cents to dollars.
26. 6594 cents to dollars.
27. 7984 cents to dollars.
28. $200.002 to mills.
29. $404.404 to mills.
30. 6 E. and $6 to cents.

ORAL PROBLEMS.

1. Jack had 25 cents, and his father gave him 50 cents. How much had he then?

2. Mr. Duane divided $2.30 equally among 5 girls. How much did each receive?

3. Out of $6.00, a man spent $2.50. How much had he left.

4. A skilled workman earned $3.50 per day. How much did he earn in 6 days?

5. A lad received for services rendered 55 cents. He spent 10 cents for a toy, 20 cents for a novel, and 15 cents for pens and ink. How much had he left?

6. How much will 5 tons of coal cost at $5.50 per ton?

7. I have one dollar to spend. I pay $\frac{1}{2}$ dollar for a book, and $\frac{1}{4}$ dollar for pens, ink, and paper. How much remains?

8. If a gentleman buys a barrel of flour for six dollars and fifty cents, and hands the seller a ten-dollar bill, how much change should he receive?

9. A man bought a horse for $150. He kept it at an expense of $60. He then sold it for $225.25. How much did he gain?

10. At the rate of $7.50 a barrel, what will $\frac{1}{5}$ of a barrel of flour cost?

11. What would be the cost of 10 yards of cloth at $2.75 per yard?

12. A fruit-vender sold 6 apples at 3 cents apiece, and 8 oranges at 5 cents apiece. If they cost $\frac{3}{10}$ of a dollar, how much did he gain?

13. Find the cost of 3 dozen copy-books, at $1.10 per dozen.

14. Henry had $\frac{3}{4}$ of a dollar and spent $\frac{1}{5}$ of a dollar. How many cents had he remaining?

15. George has \$2.625 and Henry has \$3.375. If they share their money equally, how many dollars has each?

WRITTEN PROBLEMS.

1. Jack gives \$1.62½ for a pair of shoes, 37½ cents for a penknife, and 25 cents for a baseball. How much does he pay for all?

2. A man is indebted to A., \$740.59; to B., \$36; to C., \$.985; to D., \$1.04. How much does he owe?

3. Find the sum of 19 dollars, 7 cents, 5 mills; 20 dollars, 9 cents, 9 mills; 24 dollars, 23 cents, 6 mills.

4. Mr. Rex paid for repairs as follows: Carpenter-work, \$424.30; plastering, \$170.48; plumbing, \$75.97; incidental expenses, \$205.49. How much did he pay for repairs?

5. How much must be added to \$70.039 to make the sum \$1106.39?

6. A man sold 54.6 acres of land, which cost him \$49.60 per acre, for \$3000. How much did he gain?

7. How much must you add to \$40.173 to make \$100?

8. A farmer sold 52.375 pounds of butter for \$7.856¼. How much did he receive a pound for it?

9. If one pound of butter costs 12½ cents, what will 4 firkins cost, each weighing 56 pounds?

10. If 5.3 yards of cloth cost \$9.275, what will 8.5 yards cost?

11. Bought a roll of carpet, containing 82 yards, for \$45, and sold it for 75 cents a yard. How much did I gain?

12. A shoemaker sells 35 pairs of shoes for $70.35, of which 21 pairs are sold at $2.25 a pair. At what price per pair are the rest of the shoes sold?

13. A man worked 9 days for $2.12½ per day. How much did he earn?

14. What is the value of 67.75 acres of land at $62.50 per acre?

15. When tea is $.50 per pound, how much can be bought for $.75?

16. At $1.25 per yard, how many yards of cloth can be bought for $35?

17. How many pounds of butter, at 33⅓ cents a pound, can be bought for $32?

18. A pound sterling is worth $4.8665. What are 38.8 pounds sterling worth?

19. An errand boy earns $2.75 a week. In how many weeks will he earn $49.50?

20. A man earns $12 a week and spends on an average, $8.50 a week. In how many weeks will he save $140.

21. How many quarts of berries, at $.08 a quart, will it take to pay for 4 yards of cloth at $.84?

22. A merchant sold 25.5 yards of cambric at $.20 per yard and gained $1.70. How much did it cost him?

23. If 13.5 yards of cloth cost $84⅜, what will 23.75 yards cost?

24. If I earn $70 per month and spend $45.50 of it, in how many months will I save $1080?

25. If 3 barrels of apples cost $19.125, find the cost of 100 barrels?

26. If a man spends $.87 in one day, how much will he spend in 15.5 days?

27. A grocer bought 10 barrels of sugar, each containing 235 pounds, for $152.75. How much did it cost per pound?

28. Paid $24 for cuffs at 16⅔ cents per pair. How many dozen pairs were bought?

29. Bought 8 firkins of butter for $72, and gave 6 of them for 7 yards of cloth. What was a yard of the cloth worth?

30. A load of hay, at 75 cents per 100 pounds, cost $13.98. What was the weight of the hay?

31. When ⅝ of $785 was spent, how much remained?

32. If a workman saves $62.40 in a year by laying up 20 cents each day, how long would it take 4 men at the same rate to save $124.80?

33. If 44⅔ yards of cloth cost $199, how much must be paid for 80 yards?

34. An income of 4325 dollars is spent as follows: ½ at home, and ½ of the remainder abroad. How much was spent abroad?

BILLS AND ACCOUNTS.

DEFINITIONS.

1. A **Bill** is a written statement showing the quantity and price of the items bought, together with the total cost.

2. A bill is **Receipted** when the words "Rec'd payment," written at the bottom, are followed by the signature of the maker of the bill.

3. The person who owes the bill is called the **Debtor.**

4. The person to whom the bill is owing is called the **Creditor.**

5. The **Footing** of a bill is the total amount of it.

6. An **Account** is a bill which contains both debit and credit items.

7. The **Balance of an Account** is the difference between the amounts of the debits and credits.

Abbreviations in Common Use.

@, at.	Do., the same.	Mo., month.
%, account.	Doz., dozen.	No., or №, number.
Acc't, account.	Dr., debtor.	Pay't, payment.
Bal., balance.	Fr't, freight.	Pd., paid.
Bbl., barrel.	Hhd., hogshead.	Per, by.
Bo't, bought.	Inst., this month.	Rec'd, received.
Co., company.	Int., interest.	Ult., last month.
Cr., creditor.	Lb., pound.	Yd., yard.
Cts., cents.	Mdse., merchandise.	Yr., year.

PROBLEMS.

1. A man's account at a store stands thus:

Dr.	Cr.
$4.745	$2.76½
2.62½	1.245
1.27	.62½
.45	3.45
5.28½	1.87½
10.25	5.25

What is due the merchant?

Copy, fill out, and find the footings of the following:

2.

BALTIMORE, March 1, 1898.

MR. J. B. MOORE,

Bought of WEBB & BOND:

75½	yards of Carpeting	@ $2.12½	$
37	yards of Drugget	" 1.20	
8	Rugs	" 4.16	
5	Mats	" 2.37½	
18	yards Oil-Cloth	" 1.08	
			$

3.

NEW ORLEANS, May 20, 1898.

MR. JAMES JOHNSON,

To LELONG & NOTT, *Dr.*

To	37 bbl. Pork	@ $24.35	$
"	127 bbl. Flour	" 8.15	
"	3 hhd. Molasses—169 gal.	" .43	
"	29 firkins Butter—2120 lb.	" .31	
"	3 boxes Raisins	" 4.65	
			$

4.

NEW YORK, April 1, 1898.

MR. DAVID DIXON,

To SCHENCK & VAIL, *Dr.*

1882.				
Jan.	9	To 3 Gold Watches—$124.50, $61.24, $57.18		$
"	13	" 437 pwt. Gold Chain	@ 1.15	
Feb.	3	" 35 sets Plated Tea-Service	" 43.10	
"	15	" 7 " " "	" 51.	
				$

5.

BOSTON, Jan. 1, 1870.

DANIEL CHAPMAN & Co.,

Bo't of PALMER & BROTHER:

					$	
67	pairs	Calf Boots,	@	$3.75		
108	"	Thick "	"	2.62		
75	"	Gaiters,	"	1.12		
27	"	Buskins,	"	.86		
35	"	Slippers,	"	.70		
50	"	Rubbers,	"	1.04		
					$717	93.

Rec'd Payment,

PALMER & BROTHER,
By GEO. BAKER.

6. Nov. 23, 1899. Sold to W. P. Beaux for cash: 1 No. 1 B. W. bedstead, $22.00; 1 dressing case (18 x 36 mirror), $40.00; excelsior, .40; glass case, .50; marble box, .75; 4 mats, 16 yds. @ .12.

Make out the above in bill form, and write your own name as receiver of the cash.

7. Mrs. B. K. Lex bought of Rex & Brooke 15 yds. black china silk, @ $.65; 7 yds. green henrietta cloth, @ $.50; 6 yds. navy blue serge, @ $.80; 2 felt hats, @ $.67; 2 English velour capes, @ $14.75.

8. Sold Archibald Weaver 24 sets 1⅛ in. No. 2 bed-casters, @ 13c.; 40 sets 2 in. No. 1 bed-casters, @ 17c.; 24 sets 2 in. No. 2 bedpost-casters, @ 23c.; 18 sets No. 3 plate-casters, @ 11c.; 18 sets I. H. P. W. casters, @ 13c.; 6 sets brass H. & W. casters, @ 35c.

9. Mr. David Mason bought of George Lelong & Brother 5 blank-books, @ $2.30; 7 gross Spencerian

pens, @ $1.12½; 15 B. and S. book-keeping, @ $1.75; 4 reams cap paper, @ $3.40; 20 Townsend's commercial law, @ $2.87½; 12 packs plain cards, @ 37½c.; note-paper and ink, $2.78.

10. Mrs. A. M. Williams bought of Andrew Jenkins & Son, 37 chests of green tea, @ $23.75; 42 chests of black tea, @ $17.50; 43 casks of wine, @ $99; 12 crates of Liverpool ware, @ $75; 19 barrels of Genesee flour, @ $7.00; 23 bushels of rye, @ 60c.

11. Sold Mrs. Susan Crockett 1 bbl. sugar, 245 lb., @ 3½c.; 10 lb. oatmeal, @ 2½c.; 3 lb. honey, @ 12½c.; 4 sacks flour, @ $1.35; 3 lb. raisins, @ 13c.; 7 doz. eggs, @ 15c.; 10 lb. crackers, @ 8½c.; 1 caddy Japan tea, @ 65c.; 10 lb. salt, @ 3½c.

DENOMINATE NUMBERS.

DEFINITIONS.

1. **A Denominate Number** is a concrete number whose unit is applied to measurement.

3 feet, 8 quarts, 7 days, are denominate numbers.

2. **A Simple Denominate Number** is composed of units of the same denomination.

5 pints, 27 cubic feet, are simple denominate numbers.

3. **A Compound Denominate Number** is composed of two or more denominations that have an established relation to each other.

4 feet, 6 inches; 3 bushels, 2 pecks, 1 quart, are compound denominate numbers.

LINEAR MEASURE.

Linear Measure is used in measuring length.

Table.

12 inches (in.)	= 1 foot (ft.).
3 feet	= 1 yard (yd.).
$5\frac{1}{2}$ yards, $16\frac{1}{2}$ feet	= 1 rod (rd.).
320 rods	= 1 mile (mi.).

ORAL EXERCISES.

1. How many inches in:
   1. 3 ft.? 4 ft.? 5 ft.? 6 ft.? 7 ft.? 8 ft.?
   2. $\frac{1}{2}$ ft.? $\frac{2}{3}$ ft.? $\frac{3}{4}$ ft.? $\frac{5}{6}$ ft.? $\frac{7}{8}$ ft.? $\frac{8}{9}$ ft.?
   3. 1 yd.? 3 yd.? 4 yd.? $4\frac{1}{2}$ yd.? 5 yd. 9 in.?
2. How many feet in:
   1. 3 yd.? 4 yd.? 5 yd.? 10 yd.? 25 yd.? 112 yd.?
   2. 2 rd.? 3 rd.? 4 rd.? 5 rd.? 10 rd.? 20 rd.?
   3. 24 in.? 36 in.? 60 in.? 132 in.? 144 in.? 1728 in.?
   4. $\frac{1}{3}$ yd.? $\frac{2}{3}$ yd.? $\frac{5}{3}$ yd.? $2\frac{1}{3}$ yd.? $3\frac{2}{3}$ yd.? $5\frac{3}{3}$ yd.?
3. How many yards in:
   1. 4 rd.? 8 rd.? 10 rd.? 12 rd.? 20 rd.? 25 rd.?
   2. 12 ft.? 18 ft.? 24 ft.? 36 ft.? 144 ft.? 5280 ft.?
   3. $16\frac{1}{2}$ ft.? 1 rd.? 3 ft.? 12 in.? 36 in.? 108 in.?
4. How many rods in:
   1. 1 mi.? 2 mi.? 3 mi.? 8 mi.? 10 mi.? 12 mi.?
   2. $16\frac{1}{2}$ ft.? $5\frac{1}{2}$ yd.? 33 ft.? 66 ft.? 99 ft.? 22 yd.?

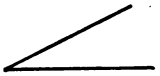
5. How many miles in 320 rd.? 640 rd.? 960 rd.? 1280 rd.? 1600 rd.?

6. $5\frac{1}{2}$ yd. = 1 rod. Compute the number of yd. in a mi.

7. $16\frac{1}{2}$ ft. = 1 rod. Compute the number of ft. in a mi.

SURFACE MEASURE.

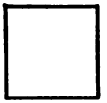
1. **Surface** or **Square Measure** is used in measuring surface. A surface has only length and breadth.



Angle.

This page at which you are looking is a surface.

2. An **Angle** is the difference in direction of two lines drawn from the same point.



Square.

3. A **Square** has four equal sides and four equal angles. The equal angles are called *right angles*.

Table.

144 Square Inches (sq. in.)	= 1 Square Foot (sq. ft.).
9 Square Feet	= 1 Square Yard (sq. yd.).
$30\frac{1}{4}$ Square Yards	= 1 Square Rod (sq. rd.).
160 Square Rods	= 1 Acre (A.).
640 Acres	= 1 Square Mile (sq. mi.).

ORAL EXERCISES.

1. How many square inches in:

1. 2 sq. ft.? 3 sq. ft.? 5 sq. ft.? 10 sq. ft.? 15 sq. ft.?

2. $\frac{1}{2}$ sq. ft.? $\frac{1}{4}$ sq. ft.? $\frac{1}{8}$ sq. ft.? $\frac{5}{6}$ sq. ft.? $\frac{5}{9}$ sq. ft.?

2. How many square feet in:

1. 5 sq. yds.? 10 sq. yds.? 25 sq. yds.? 90 sq. yds.? 100 sq. yds.?
2. 144 sq. in.? 288 sq. in.? 432 sq. in.? 720 sq. in.? 1440 sq. in.?
3. $\frac{1}{2}$ sq. yd.? $\frac{1}{3}$ sq. yd.? $\frac{2}{3}$ sq. yd.? $\frac{5}{9}$ sq. yd.? $\frac{2}{9}$ sq. yd.?

3. How many square rods in?

1. 1 A.? 2 A.? 3 A.? 4 A.? 5 A.? 10 A.?
2. $\frac{1}{2}$ A.? $\frac{1}{4}$ A.? 1 A.? $\frac{1}{16}$ A.? $\frac{3}{4}$ A.? $\frac{7}{8}$ A.?

4. How many square yards in 9 sq. ft.? 45 sq. ft.? 108 sq. ft.? 144 sq. ft.? 1728 sq. ft.?

5. How many acres in:

1. 160 sq. rds.? 320 sq. rds.? 640 sq. yds.? 1600 sq. yds.? 8000 sq. yds.?
2. $\frac{1}{2}$ sq. mi.? $\frac{1}{4}$ sq. mi.? 1 sq. mi.? $\frac{1}{16}$ sq. mi.? $\frac{3}{10}$ sq. mi.?

6. How many square feet in:

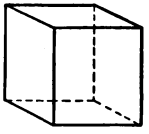
1. 3 sq. ft. and 6 ft.? 5 sq. yds. and $3\frac{1}{2}$ ft.
2. 4 sq. ft. and 72 sq. in.? 6 sq. ft. and 144 sq. in.?

VOLUME MEASURE.

1. **Volume** or **Cubic Measure** is used in measuring that which has length, breadth, and thickness.

2. The volume of a body is called its **Solid Contents**, and the body is called a **Solid**.

3. A solid with six equal square faces is called a **Cube**.



Cube.

When the faces are square inches, the solid is a *cubic inch*.

When the faces are square feet, the solid is a *cubic foot*.

Table.

1728 Cubic Inches (cu. in.)	= 1 Cubic Foot (cu. ft.).
27 Cubic Feet	= 1 Cubic Yard (cu. yd.).
128 Cubic Feet	= 1 Cord (cd.) of wood.
24¾ Cubic Feet	= 1 Perch of stone.

ORAL EXERCISES.

1. How many cubic inches in:
   1. 1 cu. ft.? 2 cu. ft.? 3 cu. ft.? 5 cu. ft.? 10 cu. ft.?
   2. $\frac{1}{2}$ cu. ft.? $\frac{1}{4}$ cu. ft.? $\frac{1}{3}$ cu. ft.? $\frac{1}{12}$ cu. ft.? $\frac{1}{144}$ cu. ft.?
2. How many cubic feet in:
   1. 1 cd.? 2 cd.? 3 cd.? 5 cd.? 10 cd.?
   2. 1 cu. yd.? 3 cu. yd.? 5 cu. yd.? 7 cu. yd.? 10 cu. yd.?
3. How many cu. yards in 27 cu. ft.? 54 cu. ft.? 81 cu. ft.? 108 cu. ft.? 2700 cu. ft.?
4. How many cu. inches in 1 cu. ft. and 144 cu. in.? 2 cu. ft. 1000 cu. in.?
5. How many cu. feet in 3 cu. yd. 5 cu. ft.? 5 cu. yd. 3 cu. ft.?

LIQUID MEASURE.

Liquid Measure is used in measuring liquids.

Table.

4 Gills (gi.)	= 1 Pint (pt.).
2 Pints	= 1 Quart (qt.).
4 Quarts	= 1 Gallon (gal.).

31½ Gallons = 1 Barrel (bbl.).
63 Gallons = 1 Hogshead (hhd.).
1 Gallon = 231 Cubic Inches.

ORAL EXERCISES.

1. How many gills in:
   1. 1 pt.? 3 pt.? 5 pt.? 8 pt.? 12 pt.?
   2. ½ pt.? ¼ pt.? ⅛ pt.? ⅛ pt.? ⅝ pt.?
2. How many pints in:
   1. 1 qt.? 3 qt.? 8 qt.? 10 qt.? 40 qt.?
   2. 3 qt. 1 pt.? 5 qt. 1 pt.? 9 qt. 2 pt.? ½ gal.? ¼ gal.?
3. How many quarts in:
   1. 1 gal.? 5 gal.? 6 gal.? 10 gal.? 25 gal.?
   2. 2 pt.? 8 pt.? 16 pt.? 20 pt.? 100 pt.?
4. How many gallons in:
   1. 4 qt.? 12 qt.? 16 qt.? 24 qt.? 40 qt.?
   2. 8 pt.? 16 pt.? 20 pt.? 56 pt.? 100 pt.?
   3. 1 bbl.? 2 bbl.? 1 hhd.? 2 hhd.?
5. How many cubic inches in a gallon? In 2 gal.?

DRY MEASURE.

Dry Measure is used in measuring grain, fruit, and vegetables.

Table.

2 Pints (pt.)	= 1	Quart (qt.).
8 Quarts	= 1	Peck (pk.).
4 Pecks	= 1	Bushel (bu.).

1 Bushel = 2150.42 cubic inches.

ORAL EXERCISES.

1. How many pints in 4 qt.? 5 qt.? 7 qt.? 8 qt.? 10 qt.?

2. How many quarts in:

   1. 2 pk.? 4 pk.? 9 pk.? 10 pk.? 25 pk.?
   2. 2 pt.? 12 pt.? 16 pt.? 20 pt.? 30 pt.?

3. How many pecks in:

   1. 1 bu.? 3 bu.? 5 bu.? 8 bu.? 40 bu.?
   2. $\frac{1}{2}$ bu.? $\frac{1}{4}$ bu.? $\frac{3}{8}$ bu.? $\frac{4}{4}$ bu.? $\frac{5}{4}$ bu.?

4. How many bushels in 4 pk.? 16 pk.? 20 pk.? 24 pk.? 36 pk.?

5. How many pecks in 8 bu. 3 pk.? In 96 qt.?

6. How many cubic inches in 1 bu.? In 2 bu.?

AVOIRDUPOIS WEIGHT.

Avoirdupois Weight is used in weighing heavy articles, except gold and silver.

Table.

16 ounces (oz.)	= 1 pound (lb.).
100 Pounds	= 1 hundred-weight (cwt.).
20 hundred-weight 2000 pounds	= 1 ton (T.).

2240 pounds = 1 long ton.

ORAL EXERCISES.

1. How many ounces in:

   1. 1 lb.? 2 lb.? 3 lb.? 4 lb.? $5\frac{1}{2}$ lb.?
   2. 1 cwt.? 10 cwt.? $\frac{1}{2}$ T.? 1 T.? $\frac{1}{10}$ T.?

2. How many pounds in:

1. 1 cwt.? 3 cwt.? $4\frac{1}{2}$ cwt.? 6 cwt.? 10 cwt.?
2. 1 T.? 3 T.? $3\frac{1}{2}$ T.? 7 T.? $10\frac{1}{2}$ T.?
3. $\frac{1}{2}$ T.? $\frac{1}{4}$ T.? $\frac{1}{5}$ T.? $\frac{3}{4}$ T.? $\frac{7}{20}$ T.?

3. How many hundred-weight in 100 lb.? 300 lb.? 400 lb.? 450 lb.? 1000 lb.?

4. How many tons in 1000 lb.? 2500 lb.? 3000 lb.? 4250 lb.? 10,000 lb.?

5. How many long tons in 2240 lb.? In 6720 lb.?

TROY WEIGHT.

Table.

24 Grains (gr.)	= 1 Pennyweight (pwt.).
20 Pennyweight	= 1 ounce (oz.).
12 ounces	= 1 pound (lb.).

1 Troy lb.	= 5760 gr.
1 Avoirdupois lb.	= 7000 gr.
1 Troy oz.	= 480 gr.
1 Avoirdupois oz.	= 437½ gr.

ORAL EXERCISES.

1. How many grains in:

1. 1 pwt.? 2 pwt.? 5 pwt.? 15 pwt.? 20 pwt.?
2. 1 oz.? 2 oz.? 3 oz.? $\frac{1}{2}$ oz.? $\frac{3}{4}$ oz.?

2. How many pennyweight in:

1. 1 oz.? 2 oz.? 6 oz.? 7 oz.? $8\frac{1}{2}$ oz.?
2. 24 gr.? 48 gr.? 72 gr.? 96 gr.? 100 gr.?

3. How many ounces in:
   1. 1 lb.? 4 lb.? 4½ lb.? 5 lb.? 6½ lb.?
   2. 20 pwt.? 40 pwt.? 50 pwt.? 60 pwt.? 70 pwt.?
   3. 3 lb. 3 oz.? 4 lb. 4 oz.? 5 lb. 5 oz.? 6 lb. 6 oz.?
4. How many pounds in 12 oz.? 24 oz.? 36 oz.? 84 oz.? 144 oz.?
5. How many gr. in 1 troy lb.? 1 avoirdupois lb.?
6. How many gr. in 1 troy oz.? 1 avoirdupois oz.?

APOTHECARIES' WEIGHT.

Apothecaries' Weight is used in weighing medicines required by prescriptions.

Table.

20 Grains (gr.)	= 1 Scruple (sc. or ℈).
3 Scruples	= 1 Dram (dr. or ʒ).
8 Drams	= 1 ounce (oz. or ℥).
12 Ounces	= 1 pound (lb. or ℔).

ORAL EXERCISES.

1. How many grains in 1 sc.? 4 sc.? 5 sc.? 5½ sc.?
2. How many scruples in:
   1. 1 dr.? 3 dr.? 6 dr.? 10 dr.? 10⅓ dr.?
   2. 20 gr.? 40 gr.? 60 gr.? 70 gr.? 110 gr.?
3. How many drams in:
   1. 1 oz.? 4 oz.? 6 oz.? 8½ oz.? 9¼ oz.?
   2. 3℈? 9℈? 24℈? 36℈? 40℈?
4. How many ounces in 1 lb.? 3 lb.? 5½ lb.? 6½ lb.? 10¾ lb.?
5. How many pounds in 12 oz.? 60 oz.? 96 oz.? 100 oz.? 150 oz.?

DIVISIONS OF TIME.

Table.

60 Seconds (sec.)	= 1 Minute (min.).
60 Minutes	= 1 Hour (hr.).
24 Hours	= 1 Day (da.).
7 Days	= 1 Week (wk.).
365 Days	= 1 Year (yr.).
366 Days	= 1 Leap Year (l. yr.).
100 Years	= 1 Century (C.).

Centennial years exactly divisible by 400, and other years exactly divisible by 4, are leap years.

12 Months = 1 Year (yr.).

Table.

1. January (Jan.)	= 31 da.	7. July (July)	= 31 da.
2. February (Feb.)	= 28 or 29* da.	8. August (Aug.)	= 31 da.
3. March (Mar.)	= 31 da.	9. September (Sept.)	= 30 da.
4. April (Apr.)	= 30 da.	10. October (Oct.)	= 31 da.
5. May (May)	= 31 da.	11. November (Nov.)	= 30 da.
6. June (June)	= 30 da.	12. December (Dec.)	= 31 da.

A Useful Rhyme.

Thirty days hath September,
April, June, and November.
All the rest have thirty-one,
Excepting February, which stands alone
With twenty-eight, till one day more
We add to it one year in four.

---

* One day added to make leap year.

ORAL EXERCISES.

1. How many seconds in 1 min.? 3 min.? 8 min.? 12 min.? 50 min.?

2. How many minutes in:

   1. 1 hr.? 2 hr.? 9 hr.? 13 hr.? 15 hr.?
   2. 60 sec.? 120 sec.? 200 sec.? 300 sec.? 500 sec.?

3. How many hours in:

   1. 1 da.? 4 da.? 10 da.? 20 da.? 30 da.?
   2. $\frac{1}{2}$ da.? $\frac{3}{4}$ da.? $\frac{5}{6}$ da.? $\frac{11}{12}$ da.? $\frac{7}{8}$ da.?
   3. 60 min.? 240 min.? 300 min.? 1200 min.? 1800 min.?

4. How many days in:

   1. 1 wk.? 7 wk.? 9 wk.? 12 wk.? 52 wk.?
   2. 3 wk. 3 da.? 4 wk. 2 da.? 4 wk. 3 da.? $\frac{1}{2}$ wk.?

5. Which of these are leap years: 1492? 1500? 1600? 1876? 1892? 1898? 1900? 2000?

6. Name the four months that have 30 da. each.

7. Name the month that changes the number of its days. Give a reason for the change.

COUNTING.

The following denominations are frequently used in counting:

Table.

12 things = 1 dozen (doz.).
12 dozen = 1 gross (gr.).
12 gross = 1 great gross (G. gr.).
20 things = 1 score.

Stationers' Table.

24 sheets	= 1 quire (qr.).
20 quires	= 1 ream (R.)
2 reams	= 1 bundle.
5 bundles	= 1 bale.

ORAL EXERCISES.

1. How many single things in:
   1. 1 doz.? 5 doz.? 12 doz.? 15 doz.? 30 doz.?
   2. $\frac{1}{2}$ doz.? $\frac{1}{4}$ doz.? $\frac{3}{4}$ doz.? $\frac{4}{4}$ doz.? $\frac{1}{6}$ doz.?
   3. 1 score? 2 score? 8 score? 10 score? 12 score?
2. How many things in:
   1. 1 gr.? 5 gr.? 9 gr.? 12 gr.? 20 gr.?
   2. $\frac{3}{5}$ score? $\frac{6}{5}$ score? $\frac{8}{5}$ score? $\frac{12}{5}$ score? $\frac{20}{5}$ score?
   3. 1 G. gr.? 3 G. gr.? 5 G. gr.? $\frac{1}{2}$ G. gr.? $\frac{3}{4}$ G. gr.?
3. How many sheets in 1 quire? 20 quires? 1 R.? 2 R.? 1 bundle? 5 bundles? 1 bale?

REVIEW.

Find the value of x in the following equations:

The sign $\therefore$ means *therefore*.

1. 3 ft. $= x$ in.

   1 ft. $= 12$ in.

   $\therefore x = 3 \times 12$ in. $= 36$.

2. 96 in. $= x$ ft.

   12 in. $= 1$ ft.

   $\therefore x$ ft. $= \frac{96}{12}$ ft. $= 8$ ft.

3. 64 qt. $= x$ pk.
4. $\frac{2}{4}$ A. $= x$ sq. rd.
5. 5 yd. $= x$ in.
6. 96 qt. $= x$ gal.
7. $\frac{10}{3}$ hr. $= x$ min.
8. 5 R. $= x$ qr.

9. 6 lb. $= x$ oz.
10. 5 bu. $= x$ pk.
11. 12 qt. $= x$ pt.
12. 7 oz. $= x$ pwt.
13. 27 sq. ft. $= x$ sq. yd.
14. 140 da. $= x$ wk.
15. 120 items $= x$ doz.
16. 6 qr. $= x$ sheets.
17. 81 cu. ft. $= x$ cu. yd.
18. $\frac{3}{4}$ cu. ft. $= x$ cu. in.
19. 144 gr. $= x$ pwt.
20. 72 oz. $= x$ lb.
21. 5 score $= x$ units.
22. 2 gross $= x$ doz.
23. $\frac{1}{2}$ G. gr. $= x$ doz.
24. \$8 $= x$ cents.
25. \$$\frac{3}{4}$ $= x$ cents.
26. 1000 mills $= \$x$.

REDUCTION OF DENOMINATE NUMBERS.

DEFINITIONS.

1. **Reduction** changes a denominate number from one denomination to another without altering its value.

2. **Reduction Descending** changes a number from a higher to a lower denomination.

3. **Reduction Ascending** changes a number from a lower to a higher denomination.

Reduction Descending.

EXERCISES.

1. How many inches are there in 8 yd. 2 ft. 5 in.?

Process.

8 yd. 2 ft. 5 in.

$\times 3$

24

$+ 2$

26

$\times 12$

312

$+ 5$

317

Explanation.

1 yd. = 3 ft.

$\therefore$ 8 yd. = 8 $\times$ 3 ft. = 24 ft.

24 ft. + 2 ft. = 26 ft.

1 ft. = 12 in.

$\therefore$ 26 ft. = 26 $\times$ 12 in. = 312 in.

312 in. + 5 in. = 317 in.

Therefore, 8 yd. 2 ft. 5 in. = 317 in.

2. Reduce to lower denominations the following:

1. 5 yd. 3 ft. 9 in.
2. 13 yd. 2 ft. 8 in.
3. 16 yd. 2 ft. 11 in.
4. 38 yd. 2 ft. 10 in.
5. 49 yd. 1 ft. 7 in.
6. 7 gal. 3 qt. 1 pt.
7. 14 gal. 2 qt. 1 pt.
8. 26 gal. 3 qt. 1 pt.
9. 29 gal. 3 qt. 1 pt.
10. 58 gal. 1 qt. 1 pt.
11. 5 hr. 26 min. 26 sec.
12. 21 hr. 23 min. 29 sec.
13. 29 hr. 30 min. 46 sec.
14. 40 hr. 40 min. 40 sec.
15. 5 cwt. 41 lb. 9 oz.
16. 9 cwt. 86 lb. 13 oz.
17. 3 T. 6 cwt. 90 lb. 13 oz.
18. 5 T. 8 cwt. 46 lb. 10 oz.
19. 10 T. 10 cwt. 10 lb. 10 oz.
20. 9 lb. 5 oz. 11 pwt. 5 gr.

3. Reduce to lower denominations the following:

1. 6 lbs. 6 oz. 17 pwt. 10 gr.
2. 11 lb. 11 oz. 11 pwt. 11 gr.
3. 4 bu. 4 pk. 6 qt. 1 pt.
4. 6 bu. 3 pk. 7 qt. ½ pt.
5. 7 bu. 2 pk. 5 qt. 1 pt.
6. 100 bu. 3 pk. 7 qt. 1 pt.
7. 5 rd. 4 yd. 2 ft. 7 in.
8. 6 rd. 3 yd. 2 ft. 2 in.
9. 9 rd. 2 yd. 2 ft. 11 in.
10. 18 rd. 1 yd. 1 ft. 1 in.
11. 4 sq. yd. 5 sq. ft. 19 sq. in.
12. 6 sq. yd. 7 sq. ft. 99 sq. in.
13. 10 sq. yd. 6 sq. ft. 141 sq. in.
14. 3 A. 120 sq. rd. 6 sq. yd.
15. 9 A. 36 sq. rd. 25 sq. yd.
16. 18 A. 72 sq. rd. 30 sq. yd.
17. 3 cu. yd. 11 cu. ft. 96 cu. in.
18. 5 cu. yd. 7 cu. ft. 825 cu. in.
19. 10 cu. yd. 10 cu. ft. 10 cu. in.

20. 4 lb. 4 oz. 4 dr. 2ʒ.
21. 5 lb. 9 oz. 6 dr. 1ʒ.
22. 10 lb. 10 oz. 7 dr. 2 sc.
23. 4 R. 11 qr. 19 sheets.
24. 5 R. 9 qr. 21 sheets.
25. 4 lb. 7 oz. 11 pwt.
26. 6 lb. 11 oz. 4 sc.
27. 7 da. 11 hr. 36 min.
28. 9 rd. 5 yd. 2 ft. 11 in.
29. 6 A. 151 sq. rd. 29 sq. yd.
30. 8 mi. 211 rd. 16 ft. 6 in.

4. Reduce .85 yd. to feet and inches.

Process.	Explanation.
.85 yd.	1 yd. = 3 ft.
3	.85 yd. = .85 of 3 ft. = 2.55 ft.
2.55 ft.	1 ft. = 12 in.
12	.55 ft. = .55 of 12 in. = 6.6 in.
6.60 in.	∴ .85 yd. = 2 ft. 6.6 in.

5. $\frac{5}{8}$ cwt. equal how many pounds and ounces?

Process.	Explanation.
$\frac{5}{8} \times 100 = \frac{500}{8} = 62\frac{1}{2}$.	1 cwt. = 100 lb.
$\frac{1}{2} \times 16 = \frac{16}{2} = 8$.	$\frac{5}{8}$ cwt. = $\frac{5}{8}$ of 100 lb. = $62\frac{1}{2}$.
∴ $\frac{5}{8}$ cwt. = 62 lb. 8 oz.	1 lb. = 16 oz.
	$\frac{1}{2}$ lb = $\frac{1}{2}$ of 16 oz. = 8 oz.
	∴ $\frac{5}{8}$ cwt. = 62 lb. 8 oz.

Or,

$\frac{5}{8}$ = .625	$\frac{5}{8}$ cwt. = .625 cwt.
100	1 cwt. = 100 lb.
62.500	.625 cwt. = .625 of 100 lb. = 62.5 lb.
16	1 lb. = 16 oz.
8.000	.5 lb. = .5 of 16 oz. = 8 oz.

∴ $\frac{5}{8}$ cwt. = 62 lb. 8 oz.

6. Reduce to integers of lower denominations:

1. $\frac{3}{8}$ rd.
2. $\frac{5}{9}$ wk.
3. $\frac{4}{7}$ yd.
4. .7 bu.
5. .56 T.
6. .5 cu. yd.
7. $\frac{1}{6}$ mi.
8. $\frac{4}{7}$ A.
9. .84 wk.
10. .4236 gal.
11. .585 yd.
12. .625 bundle.
13. $\frac{7}{9}$ G. gr.
14. $\frac{5}{9}$ lb. (Apoth.)
15. $\frac{7}{10}$ lb. (Troy.)
16. $\frac{8}{9}$ T.
17. $\frac{2}{8}$ bu.
18. $\frac{7}{8}$ gal.
19. .3456 cd.
20. .1234 A.
21. $\frac{11}{12}$ mi.
22. $\frac{3}{8}$ sq. mi.
23. $\frac{5}{12}$ cu. yd.
24. .875 gal.
25. .4 bu.
26. .0$\frac{4}{5}$ oz. (Troy.)
27. .001 lb. (Apoth.)
28. .0007 C. (Time.)
29. .9009 score.
30. $\frac{144}{1728}$ bale.

Reduction Ascending.

EXERCISES.

1. Reduce 345 pints to bushels, pecks and quarts.

Process.

2 | 345 pt.
8 | 172 qt. 1 pt.
4 | 21 pk. 4 qt.
5 bu. 1 pk.

$\therefore$ 345 pt. = 5 bu. 1 pk. 4 qt. 1 pt.

Explanation.

2 pt. = 1 qt. $\therefore$ $\frac{1}{2}$ the number of pints = the number of quarts.

345 pt. = $\frac{345}{2}$ qt. = 172 qt. 1 pt.

8 qt. = 1 pk. $\therefore$ $\frac{1}{8}$ the number of quarts = the number of pk.

172 qt. = $\frac{172}{8}$ pk. = 21 pk. 4 qt.

4 pk. = 1 bu. $\therefore$ $\frac{1}{4}$ the number of pecks = the number of bu.

21 pk. = $\frac{21}{4}$ bu. = 5 bu. 1 pk.

Hence, 345 pt. = 5 bu. 1 pk. 4 qt. 1 pt.

2. Reduce 4392 inches to rods, yards, etc.

Process.

12 |	4392 in.
3 |	366 ft. 0 in.
$5\frac{1}{2}$ |	122 yd. 0 ft.
2 |	2
11 |	244 half yd.
	22 rd. 2 half yd.
	or 1 yd.

$\therefore$ 4392 in. = 22 rd. 1 yd.

Explanation.

12 in = 1 ft. $\therefore \frac{1}{12}$ the number of in. = the number of feet.

4392 in. = 366 ft.

3 ft. = 1 yd. $\therefore \frac{1}{3}$ the number of feet = the number of yd.

366 ft. = $\frac{366}{3}$ yd. = 122 yd.

$5\frac{1}{2}$ yd. = 11 half yd. = 1 rd.

$\therefore \frac{1}{11}$ the number of half yd. = the number of rods.

$\frac{244}{11}$ yd. = $\frac{244}{11}$ rd. = 22 rd. 2 half yd.

2 half yd. = 1 yd.

Hence, 4392 in. = 22 rd. 1 yd.

3. Reduce:

1. 5324 gi. to gal., etc.
2. 4296 pt. to gal., etc.
3. 6835 gi. to gal., etc.
4. 4640 in. to rd., etc.
5. 3567 in. to rd., etc.
6. 2706 in. to rd., etc.
7. 4675 pt. to bu., etc.
8. 5794 pt. to bu., etc.
9. 4058 pt. to bu., etc.
10. 6275 oz. to cwt., etc.
11. 9238 oz. to cwt., etc.
12. 6094 lb. to tons, etc.
13. 4806 lb. to tons, etc.
14. 5396 hr. to wk., etc.
15. 9279 sec. to hr., etc.
16. 10,905 min. to da., etc.

4. Reduce:

1. 4054 sq. in. to sq. yd., etc.
2. 6048 sq. in. to sq. yd., etc.
3. 2905 sheets to R., etc.
4. 3426 sheets to R., etc.
5. 8975 cu. in. to cu. ft., etc.
6. 7865 gr. to lb. (troy), etc.

7. 9497 gr. to lb. (troy), etc.
8. 10,249 gr. to lb. (apoth.), etc.
9. 876,000 hr. to C.
10. 392,040 sq. ft. to A.

5. Reduce 6 oz. 6 pwt. to the fraction of 1 lb.

Process.		Explanation.
6 oz. 6 pwt.	1 lb.	1 lb. $= 12 \times 20 = 240$ pwt.
20	12	6 oz. 6 pwt. $= 126$ pwt.
126 pwt.	12 oz.	1 pwt. $= \frac{1}{240}$ of 1 lb.
	20	$\therefore$ 126 pwt. $= \frac{126}{240}$ of 1 lb. $= \frac{21}{40}$ of 1 lb.
$\frac{126}{240} = \frac{21}{40}$.	240 pwt.	Hence, 6 oz. 6 pwt. $= \frac{21}{40}$ of a lb.

6. Reduce:
   1. 2 ft. 4 in. to the fraction of a yard.
   2. 4 qt. 1 pt. to the fraction of a peck.
   3. 5 pk. 4 qt. to the fraction of a bushel.
   4. 3 qt. 1 pt. 3 gi. to the fraction of a gallon.
   5. 3 pk. 5 qt. 1 pt. to the fraction of a bushel.
   6. 5 yd. 2 ft. 7 in. to the fraction of a rod.
   7. 5 da. 6 hr. to the fraction of a week.
   8. 15 hr. 12 min. 18 sec. to the fraction of a day.
   9. 2 cwt. 5 lb. 12 oz. to the fraction of a ton.
   10. 5 qt. 1 pt. to the fraction of a bushel.
   11. 3 gal. 4 qt. 1 pt. 3 gi. to the fraction of a barrel.
   12. $5 2 cents 5 mills to the fraction of an eagle.
   13. 6 oz. 6 pwt. 6 gr. to the fraction of a lb. Of 2 lb. Of 5 lb.
   14. 360 da. 3 wk. 3 da. 4 hr. to the fraction of a year. Of 10 years.

ADDITION OF DENOMINATE NUMBERS.

EXERCISES.

1. Find the sum of 4 gal. 4 qt. 1 pt.; 6 gal. 3 qt. 1 pt.; 7 gal. 3 qt. 1 pt.; 9 gal. 2 qt.

Process.

Gal.	Qt.	Pt.
4	4	1
6	3	1
7	3	1
9	2	0
29	1	1

Explanation.

1. PRINCIPLE: *Only units of like order can be added.*

2. Numbers of like denomination are written in the same column.

3. The sum of the pints is 3 pt. = 1 qt. 1 pt.

4. We write the 1 pt. and reserve the 1 qt.

5. The sum of the 1 qt. and the column of qt. is 13 qt. = 3 gal. 1 qt.

6. The sum of the 3 gal. and the column of gal. is 29 gal.

Hence, the sum total is 29 gal. 1 qt. 1 pt.

2. Find the sum of:

1. 4 gal. 3 qt. 1 pt.; 28 gal. 2 qt. 1 pt.; 29 gal. 3 qt.
2. 9 bu. 2 pk. 6 qt.; 27 bu. 3 pk. 6 qt.; 23 bu. 2 pk. 5 qt.
3. 8 da. 6 hr. 31 min.; 9 da. 25 hr. 21 min.; 7 da. 29 hr.
4. 35 lb. 7 oz. (avoirdupois); 46 lb. 15 oz.; 37 lb. 13 oz.; 94 lb.
5. 24 lb. 5 oz. 19 pwt. 9 gr.; 22 lb. 6 oz. 18 pwt. 21 gr.; 21 lb. 21 gr.
6. 21 yd. 2 ft. 11 in.; 26 yd. 2 ft. 10 in.; 9 yd. 1 ft. 8 in.; 29 yd. 2 ft. 9 in.
7. 26 gal. 2 qt. 1 pt. 2 gi.; 29 gal. 3 qt. 1 pt. 1 gi.; 39 gal. 3 qt.; 32 gal.
8. 23 A. 46 sq. rd.; 25 A. 120 sq. rd.; 26 A. 143 sq. rd.; 22 A. 107 sq. rd.

9. 6 T. 7 cwt. 25 lb. 11 oz.; 8 T. 6 cwt. 47 lb. 14 oz.; 28 T. 95 lb.

10. 6 yd. 1 ft. 9 in.; 5 yd. 2 ft. 9 in.; 5 yd. 1 ft. 11 in.; 4 yd. 2 ft.; 4 yd.

SUBTRACTION OF DENOMINATE NUMBERS.

1. From 26 gal. 2 qt. 1 pt. 2 gi. take 19 gal. 3 qt. 0 pt. 3 gi.

Process.

	Gal.	Qt.	Pt.	Gi.
	26	2	1	2
	19	3	0	3
Rem.	6	3	0	3

Explanation.

1. PRINCIPLE: *Only like numbers can be subtracted.*

2. We write the numbers of the same denomination in the same column.

3. We begin to subtract at the lowest denomination.

4. 3 gi. cannot be taken from 2 gi.

5. We therefore add the 1 pt., or 4 gi., to the 2 gi., making 6 gi., and then say 6 gi. — 3 gi. = 3 gi.

6. The 1 pt. having been used, 0 pt. — 0 pt. = 0 pt.

7. 3 qt. cannot be taken from 2 qt.

8. Adding 1 gal., or 4 qt., we have 6 qt.

9. 6 qt. — 3 qt. = 3 qt.

10. Finally, 25 gal. — 19 gal. = 6 gal.

11. Remainder = 6 gal. 3 qt. 0 pt. 3 gi.

2. From 9 bu. 3 pk. 5 qt. take 4 bu. 3 pk. 6 qt.

3. From 9 bu. 0 pk. 4 qt. take 4 bu. 3 pk. 6 qt.

4. From 13 gal. 3 qt. 1 pt. 3 gi. take 6 gal. 4 qt. 0 pt.

5. From 11 da. 6 hr. 31 min. take 8 da. 8 hr. 11 min.

6. From 42 lb. 5 oz. 16 pwt. take 9 lb. 7 oz. 13 pwt.

7. From 26 yd. 1 ft. 8 in. take 5 yd. 2 ft. 11 in.

8. From 23 rd. 5 yd. 2 ft. take 9 rd. 5 yd. 2 ft.

9. From 24 lb. 9 oz. 6 dr. take 8 lb. 11 oz. 6 dr.

10. From 81 sq. rd. take 26 sq. rd. 7 sq. ft. 111 sq. in.

11. From 36 T. 9 cwt. 86 lb. 11 oz. take 21 T. 13 cwt. 46 lb. 13 oz.

12. From 132 gal. 1 qt. 1 pt. 1 gi. take 128 gal. 3 qt. 1 pt. 3 gi.

DIFFERENCE OF DATES.

1. How many years, months, and days from June 27, 1853, to Dec. 25, 1898?

Process.

Yr.	Mo.	Da.
1898	12	25
1853	6	27
45	5	28

Explanation.

1. Dec. is the 12th mo., June the 6th mo.

2. We add 1 mo. or 30 da. to 25 da., making 55 days.

3. We then subtract, proceeding as with other denominate numbers.

2. How long was it between:

1. Jan. 21, 1846, and June 26, 1898?
2. July 24, 1875, and Feb. 2, 1891?
3. Mar. 9, 1840, and Aug. 8, 1892?
4. Oct. 24, 1882, and Sept. 11, 1893?
5. Jan. 6, 1706, and Apr. 17, 1790?

MULTIPLICATION OF DENOMINATE NUMBERS.

1. Multiply 7 yd. 2 ft. 9 in. by 6.

Process.

Yd.	Ft.	In.
7	2	9
		6
47	1	6

Explanation.

1. 6 times 9 in. = 54 in. = 4 ft. 6 in.

2. We write the 6 in. and reserve the 4 ft.

3. 6 times 2 ft. = 12 ft.; 12 ft. + 4 ft. reserved = 16 ft. = 5 yd. 1 ft.

4. We write the 1 ft. and reserve the 5 yd.

5. 6 times 7 yd. = 42 yd.; 42 yd. + 5 yd. = 47 yd.

Hence, the product is 47 yd. 1 ft. 6 in.

2. Multiply:

1. 5 bu. 2 pk. 6 qt. by 8.
2. 6 gal. 2 qt. 1 pt. 3 gi. by 7.
3. 6 lb. 7 oz. 11 pwt. 9 gr. by 9.
4. 8 lb. 9 oz. 6 dr. 2ϑ 11 gr. by 8.
5. 4 hr. 30 min. 46 sec. by 6.
6. 3 T. 6 cwt. 59 lb. 14 oz. by 9.
7. 3 rd. 4 yd. 2 ft. 11 in. by 10.
8. 23 cu. yd. 16 cu. ft. 1226 cu. in. by 7.
9. 9 sq. yd. 3 sq. ft. 56 sq. in. by 6.
10. 6 da. 9 hr. 26 min. 36 sec. by 5.
11. 23 rd. 5 yd. 2 ft. 9 in. by 9.
12. 6 R. 9 qr. 17 sheets by 10.
13. 7 bbl. 11 gal. 2 qt. 1 pt. by 8.
14. 6 yr. 5 mo. 15 da. 18 hr. by 5.
15. 6 T. 12 cwt. 95 lb. 12 oz. 9 dr. by 8.

DIVISION OF DENOMINATE NUMBERS.

1. Divide 45 bu. 3 pk. 1 qt. by 8.

Process.

	bu.	pk.	qt.
8)	45	3	1
	5	2	$7\frac{1}{8}$

Explanation.

1. $\frac{1}{8}$ of 45 bu. = 5 bu. and 5 bu. remaining.
2. 5 bu. rem. = 20 pk.; 20 pk. + 3 pk. = 23 pk.
3. $\frac{1}{8}$ of 23 pk. = 2 pk., and 7 pk. remaining.
4. 7 pk. = 56 qt.; 56 qt. + 1 qt. = 57 qt.
5. $\frac{1}{8}$ of 57 qt. = $7\frac{1}{8}$ qt.

Hence, the quotient is 5 bu. 2 pk. $7\frac{1}{8}$ qt.

2. Divide:

1. 32 gal. 2 qt. 1 pt. 2 gi. by 7.
2. 26 bu. 3 pk. 6 qt. 1 pt. by 5.
3. 24 yd. 2 ft. 7 in. by 8.
4. 34 cwt. 79 lb. 11 oz. by 6.

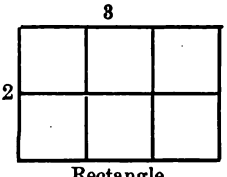
5. 53 lb. 9 oz. 16 pwt. by 10.
6. 33 lb. 8 oz. 6 dr. 2ϑ by 9.
7. 20 hr. 11 min. 47 sec. by 7.
8. 32 sq. yd. 8 sq. ft. 56 sq. in. by 5.
9. 26 rd. 5 yd. 2 ft. 9 in. by 6.
10. 16 bbl. 11 gal. 2 qt. 1 pt. by 8.
11. 144 bu. 2 pk. 6 qt. by 7.
12. 42 gal. 1 qt. 1 pt. 1 gi. by 12.
13. 17 mi. 100 rd. 13 ft. 6 in. by 11.
14. 15 A. 140 sq. rd. 3 sq. yd. by 3.
15. 12 T. 5 cwt. 80 lb. 12 oz. by 7.

MEASUREMENT OF SURFACES.

1. A square has been defined.

2. Describe a square inch. Describe a square foot.

3. A figure 2 inches long and 1 inch wide, with all its angles equal, contains how many square inches?



Rectangle.

4. A figure 2 inches long and 2 inches wide contains how many square inches?

5. That is 2, the length, × 2, the width, = what?

6. A figure 3 in. long and 2 in. wide contains how many square inches?

7. That is 3, the length, × 2, the width, = what?

8. The number of square units in a figure equals the product of what?

9. The number of square units in a figure is called its **Area.**

10. A figure with four sides and four right angles is called a **Rectangle.**

PRINCIPLE.

The area of a rectangular surface is the product of its length and width.

FORMULA.

Area of a Rectangle = Length × Width.

PROBLEMS.

1. Find the area of a rectangular lot whose length is 33 ft. and width 6 ft. 6 in.

Process.	Explanation.
$33 \times 6\frac{1}{2} = 214\frac{1}{2}$.	Area = Length × Width.
$\therefore$ Area $= 214\frac{1}{2}$ sq. ft.	The product of 33 and $6\frac{1}{2}$ is $214\frac{1}{2}$.
	Hence, the area is $214\frac{1}{2}$ sq. ft.

The process requires that the length and width be expressed in the same denomination.

2. A room is 24 ft. long and 18 ft. wide. What is its area?

3. A floor is 10 ft. wide and 30 ft. long. Find its area.

4. The side wall of a room is 17 ft. long and 12 ft. high. How many square feet in the wall? How many square yards?

5. An end wall of a room is 12 ft. by 12 ft. How many square feet in the wall? How many square yards?

6. I have a rectangular room. Its length is 20 ft., its width, 18 ft.; its height, 12 ft. Find the number of square feet and square yards in the four walls.

Suggestion: The distance around the room × the height = the area.

7. A rectangular field is, in length, 150 rods; in breadth, 130 rods. Find the number of square rods in the field and also the number of acres.

8. A room is 25 ft. long and 15 ft. wide. Find the area of both ceiling and floor; also the cost of plastering the ceiling, at 25 cents per square yard.

9. Find the cost of painting the walls and ceiling of a room 16 ft. 6 in. long, 15 ft. 9 in. wide, 14 ft. high, at 25 cents per square yard.

10. Find the area of a square field whose side is 76 rods.

11. Find the value of a field 180 rds. long and 90 rds. wide, at $20 an acre.

12. A room 18 ft. wide and 24 ft. long was covered with carpet, 1 yd. wide, at $1.00 per yard. How much did the carpeting cost?

13. I have a rectangular field 180.5 rds. long and 97.75 rds. wide. How many acres does it contain?

14. What will be the cost of cementing the floor of a cellar 56 ft. by 45 ft., at $.30 per square yard?

15. What will it cost to pave a street 3 mi. 115 rds. long and 2 rds. wide, at $46.50 per square rod?

16. How many square rods in a garden that is 7 rods square?

17. How many acres in a rectangular field 36 rds. 12 ft. wide, and 48 rds. 8 ft. long?

18. What is the cost of asphalting a walk $93\frac{1}{2}$ ft. by $6\frac{1}{2}$ ft., at $.75 per square yard?

19. A side of a public square is 660 ft. How many acres does it contain?

20. If a carpet is 36 in. wide, how many yards of it

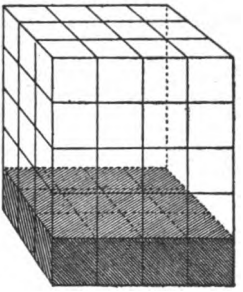
will be required to carpet a room 18 ft. by 16 ft? What will be the cost at \$1.12½ per yard?

21. Draw a figure to show the difference between 6 square feet and 6 feet square.

MEASUREMENT OF SOLIDS.

A cube has been defined.

Describe a cubic inch. A cubic foot. A cubic yard.



How many squares in the base of the figure?

We will call them square feet.

If you place a cubic foot of wood or stone upon each square, how many cubic feet will you have?

If upon those you place 12 more cubic feet, you will have how many cubic feet?

If you add a third layer of 12 cubic feet, how many will you have? If you add a fourth layer, how many? If you add a fifth layer, how many?

How many feet high is your structure now?

The base layer has how many cubes?

Therefore, the number 12, the base, × 5, the height, gives you what?

A **Rectangular Solid** stands on a rectangular base and its angles are all right angles.

The **Volume** of a solid is the number of cubic units it contains, *i.e.*, of cubic inches, cubic feet, cubic yards.

PRINCIPLE.

The volume of a rectangular solid is the product of its length, width, and height.

FORMULA.

Volume of a Rectangular Solid = Length × Width × Height.

EXERCISES.

1. How many cubic feet in a pile of wood 4 ft. high, 8 ft. long, and 4 ft. wide?

Process.

Base. h. v.
$(8 \times 4) \times 4 = 32 \times 4 = 128$

Explanation.

1. Volume = length × width × height.
2. ∴ The volume = 8 × 4 × 4 = 128 (cubic feet).

The factors of the volume must be expressed in the same denomination.

2. Find the volume of the following solids:

	Length.	Width.	Height.	Thickness.
1.	20 ft.	8 ft.	——	7 ft.
2.	10 in.	9 in.	12 in.	——
3.	9 yd.	6 yd.	10 yd.	——
4.	15 ft.	11 ft.	——	10 ft.
5.	9 ft. 6 in.	7 ft.	10 ft.	——
6.	13 ft. 4 in.	10 ft.	——	9 ft.
7.	11 ft. 8 in.	11 ft.	13 ft.	——
8.	13 ft. 3 in.	12 ft.	——	9 ft.
9.	9 ft. 10 in.	9 ft. 6 in.	——	9 ft.
10.	25 ft. 9 in.	25 ft.	100 ft.	——

3. Find the value of the cords of wood in the following piles:

	Length.	Width.	Height.	Cost per cord.
1.	35 ft.	4 ft.	8 ft.	$3.00
2.	43 ft.	4 ft.	16 ft.	$3.25
3.	59 ft.	27 ft.	13 ft.	$3.50
4.	144 ft.	12 ft.	16 ft.	$4.00
5.	200 ft.	25 ft.	25 ft.	$5.50

4. Find the cost of the following stone piles:

	Height.	Length.	Thickness.	Cost per perch.
1.	8 ft.	50 ft.	2 ft.	$2.25

A perch = 24.75 cu. ft.

Suggestion: $\frac{8 \times 50 \times 2 \times 2.25}{24.75}$.

2.	9 ft.	75 ft.	2½ ft.	$2.50
3.	5 ft.	112 ft.	18 in.	$4.00
4.	120 ft.	20 ft.	20 ft.	$3.00
5.	30 ft.	8 ft. 9 in.	2 ft.	$ .75

5. Find the cost of excavating the following cellars:

	Length.	Width.	Depth.	Cost per cubic yd.
1.	40 ft.	20 ft.	8 ft.	30 cents.
2.	30 ft.	20 ft.	6 ft.	25 cents.
3.	38 ft.	30 ft.	8 ft.	45 cents.
4.	56½ ft.	40½ ft.	8½ ft.	50 cents.
5.	40 ft. 6 in.	30 ft. 9 in.	8¼ ft.	40 cents.

BOARD MEASURE.

1. In measuring lumber, a board *1 inch thick or less* is treated as a mere surface.

2. A board 1 ft. wide, 12 ft. long, and 1 inch thick is bought or sold as containing 12 ft. *board measure.*

3. Thickness, however, becomes a factor when it exceeds 1 inch.

FORMULA.

Length (ft.) × Width (ft.) × Thickness (in.) = Feet Board Measure.

PROBLEMS.

1. How many feet, board measure, are there in a timber 30 ft. long, 7 in. wide, and 6 in. thick?

Process.

$$\frac{\cancel{30}^{15}}{1} \times \frac{7}{\cancel{12}_{2}} \times \frac{\cancel{6}}{1} = 105 \text{ ft.}$$

Explanation.

1. 7 in. $= \frac{7}{12}$ ft.

2. $\frac{15}{1} \times \frac{7}{12}$ = the number of board feet, thickness 1 inch.

3. $\frac{15}{1} \times \frac{7}{12} \times \frac{6}{1}$ = the number of board feet, thickness 6 in.

4. By cancelling, we have 105 ft., board measure.

2. Find the number of board feet in the following pieces of lumber:

1. 16 ft. by 8 in.
2. 15 ft. by 9 in.
3. 12 ft. by 10 in.
4. 10 ft. by 12 in.
5. 14 ft. by 18 in.
6. 15 ft. by 12 in. by 6 in.
7. 40 ft. by 10 in. by 10 in.
8. 24 ft. by 9 in. by 12 in.
9. 40 ft. by 10 in. by $\frac{1}{2}$ in.
10. 12 ft. by 6 in. by $\frac{3}{4}$ in.

3. Find the cost of a dozen boards, each 15 ft. long and 12 in. wide, at $18 per M. feet.

4. I have 30 joists 20 ft. long, 18 in. wide, 4 in. thick. How many feet, board measure, in them? Find their cost at $20 per M.

5. Find the weight of a plank 15 ft. 9 in. long, 10 in. wide, and 2 in. thick, at $3\frac{7}{16}$ lb. per board foot.

6. Find the cost of 17 planks, $14\frac{1}{2}$ ft. long, 10 in. wide, and 4 in. thick, at $\$18\frac{1}{4}$ per M., board measure.

7. How many board feet in a stick of timber 46 ft. long, 10 in. thick, 12 in. wide at one end and 9 in. wide at the other end?

Suggestion: Use one-half the sum of the end widths.

PERCENTAGE.

The decimal fractions of most general importance have for their denominator 100; as, $\frac{1}{100} = .01$, $\frac{5}{100} = .05$, $\frac{25}{100} = .25$.

These fractions have given rise to **Percentage**, which means *computation by hundredths.*

Per Cent. means *by the hundred;* its symbol is %.

5 per cent. is written thus: 5%. $5\% = \frac{5}{100} = \frac{1}{20} = .05$.

To find 5% of \$200 we must take $\frac{5}{100}$ or $\frac{1}{20}$ or .05 of \$200.

$\frac{5}{100}$ of \$200 = how many dollars?
$\frac{1}{20}$ of \$200 = how many dollars?
.05 of \$200 = how many dollars?

The \$10 is called the **Percentage.**

The \$200 is called the **Base.**

The 5% is called the **Rate.**

\$200 + \$10 = \$210 is called the **Amount.**

\$200 — \$10 = \$190 is called the **Difference.**

PRINCIPLE.

The base multiplied by the rate equals the Percentage.

EXERCISES.

1. 15 per cent. $= 15\% = \frac{15}{100} = \frac{3}{20} = .15$.

Express in like manner 10 per cent. in four different forms. Also, 20 per cent., 25 per cent., 50 per cent., 75 per cent., 100 per cent., 125 per cent., 250 per cent.

2. $33\frac{1}{3}$ per cent. $= \frac{33\frac{1}{3}}{100} = \frac{100}{300} = \frac{1}{3} = .33\frac{1}{3}$.

Treat in like manner $12\frac{1}{2}$ per cent., $6\frac{1}{4}$ per cent., $62\frac{1}{2}$ per cent., $37\frac{1}{2}$ per cent., $87\frac{1}{2}$ per cent., $66\frac{2}{3}$ per cent., $17\frac{1}{2}$ per cent., $2\frac{1}{2}$ per cent., $7\frac{1}{2}$ per cent., 2 per cent., $16\frac{2}{3}$ per cent.

3. $\frac{1}{2}$ per cent. $= \frac{\frac{1}{2}}{100} = \frac{1}{200} = .005$.

Show in like manner the values of $\frac{1}{4}$ per cent., $\frac{3}{4}$ per cent., $\frac{1}{8}$ per cent., $\frac{5}{8}$ per cent., $\frac{4}{5}$ per cent.

WRITTEN EXERCISES AND PROBLEMS.

1. What is 6% of \$535.06?

Process.	Explanation.
\$535.06	1. $6\% = \frac{6}{100} = \frac{3}{50} = .06$.
.06	2. To take 6% of \$535.06, we multiply it by .06, and obtain for the percentage \$32.10 +.
\$32.1036	

2. What is 75% of \$800?

Process.	Explanation.
$\frac{3}{4}$ of \$800 $= \frac{3}{4}$ of \$800 $=$ \$600.	1. $75\% = \frac{75}{100} = \frac{3}{4} = .75$.
	2. By cancellation $\frac{3}{4}$ of \$800 $= 3$ times \$200 $=$ \$600.

3. $\therefore$ 75% of \$800 is \$600.

NOTE.—In your processes use the form of the rate that will give you the percentage most readily.

3. Find the percentage from the following bases and rates:

	Base.	Rate.		Base.	Rate.
1.	$600,	1%.	11.	$6.50,	6%.
2.	$700,	2%.	12.	$8.95,	7%.
3.	$800,	3%.	13.	$10.60,	8%.
4.	$1000,	4%.	14.	$5.25,	10%.
5.	$1250,	5%.	15.	$25.06,	12%.
6.	275 sheep,	28%.	16.	225 pounds,	25%.
7.	4380 bushels,	45%.	17.	3628 pounds,	39%.
8.	10,000 barrels,	12½%.	18.	8250 apples,	88%.
9.	1200 men,	1½%.	19.	4440 dollars,	100%.
10.	320 horses,	6¼%.	20.	1898 years,	33⅓%.

4. A grocer paid $356 for sugar and sold it at a gain of 15%. What was his gain?

5. I bought 46 cords of wood, at $3.50 per cord, and sold it at a gain of 25%. What was my whole gain?

6. What will 50 sheep cost if 75 are worth $375, and for how much must they be sold to gain 12½%?

7. I sold 500 bushels of corn for $200. The corn cost me 25% more. How much did I lose on each bushel?

8. A man bought 2165 bushels of wheat for $1515.50. A part receiving damage, he is willing to lose 10%. Find the selling price.

9. A miller charges 5% for grinding. How many quarts will he take when he has ground 35 bushels?

10. Ninety per cent. of a class of 60 pupils are promoted. How many are not promoted?

11. A girl spelled correctly 99% of 200 words. How many did she miss?

12. A lot is sold for $1050, of which is 20% is profit. What did the lot cost?

13. How much did I gain on a house for which I paid $9870, my profit being 3%.

14. A piano, marked $750, was sold for cash at 35% less. What was the selling price?

15. A man invested 12½% of $15,320 in stocks. How much did he invest?

INTEREST.

1. **Interest** is money paid for the use of money. It is computed at a rate per cent. of the money used.

2. The money used is called the **Principal** (Base).

3. The Principal × Rate = One year's interest (Percentage).

4. Interest for a greater or less time than one year = Principal × Rate × Years, or fraction of a year.

What is 6% of $500? Of $1000?

What is the interest of $500 for 1 yr., at 6%? For 2 yrs.? For 5 yrs.? For 10 yrs.? For 25 yrs.?

What is the interest of $1000 at 6% for ½ yr.? ¼ yr? ⅔ yr.? 9 mo.? 10 mo.?

Percentage, or one year's interest, multiplied by time, expressed in years, gives you what?

What are the three factors of interest?

PRINCIPLE.

The factors of interest are principal, rate, and time (years).

FORMULA.

Int. = Prin. × R. × Yr.

1. Find the interest of \$1200 for 2 yrs., at 5%.

Process.	Explanation.
\$1200 × .05 = \$60.	1. 5% of \$1200 = \$60, the int. for 1 yr.
\$60 × 2 = \$120.	2. \$60 × 2 = \$120, the int. for 2 yrs.
	3. ∴ the int. of \$1200 at 5% for 2 yrs. = \$120.

2. What is the interest of:
   1. \$250 for 1 yr. at 6%?
   2. \$300 for 1 yr. at 5%?
   3. \$500 for 1 yr. at 4%?
   4. \$600 for 1 yr. at 3%?
   5. \$1000 for 1 yr. at 7%?
   6. \$275 for 2 yrs. at 8%?
   7. \$236 for 3 yrs. at 5%?
   8. \$1673 for 4 yrs. at 6%?
   9. 430.87 for 5 yrs. at 7%?
   10. \$2846 for 6 yrs. at 8%?
   11. \$12.46 for $1\frac{1}{2}$ yrs. at 9%?
   12. \$126.37 for $\frac{1}{2}$ yr. at 7%?
   13. \$876 for $\frac{1}{4}$ yr. at 8%?
   14. \$327 for $\frac{2}{3}$ yr. at 6%?
   15. \$56.79 for $5\frac{1}{2}$ yrs. at 4%?
   16. \$182.34 for $2\frac{2}{3}$ yrs. at 6%?
   17. \$182.34 for 2 yrs. 6 mo. at 6%?
   18. \$432.81 for 1 yr. 9 mo. at 5%?
   19. \$3843 for 3 yrs. 3 mo. at 7%?
   20. \$129.95 for 1 yr. 8 mo. at 10%?

3. Find the interest of \$530.60 for 6 yr. 8 mo. at 6%.

Process.

$$\frac{\overset{106.12}{\cancel{\$530.60}} \times \overset{2}{\cancel{6}} \times \overset{4}{\cancel{80}}}{\underset{2}{\cancel{12}} \times \underset{5}{\cancel{100}}} = \$212.24.$$

Explanation.

1. $6\% = \frac{6}{100}$. 6 yr. 8 mo. = 80 mo.

2. Multiplying the principal by $\frac{6}{100}$, the rate, we get the interest for 1 year. By dividing the interest for 1 year by 12, we get the interest for 1 mo. Then multiplying the interest for 1 mo. by 80, the number of months, we get the interest required. The process is shortened by cancellation.

1. \$523.30 for 3 yrs. 9 mo. at 5%.
2. \$429.47 for 4 yrs. 7 mos. at 7%.
3. \$536.35 for 5 yrs. 6 mos. at 6%.
4. \$796.70 for 4 yrs. 8 mos. at 8%.
5. \$548.29 for 6 yrs. 3 mos. at 9%.
6. \$415.38 for 5 yrs. 5 mos. at 4%.
7. \$847.27 for 3 yrs. 10 mos. at 7%.
8. \$1046.60 for 2 yrs. 3 mos. at 6%.
9. \$858.94 for 3 yrs. 5 mos. at 6%.
10. \$1072.70 for 1 yr. 11 mos. at 6%.

4. Find the interest of \$635.48 for 3 yr. 6 mo. 18 da. at 4%.

Process.

$$\frac{\overset{6.3548}{\cancel{\$635.48}} \times \overset{14.2}{\cancel{4}} \times \overset{}{\cancel{42.6}}}{\underset{3}{\cancel{12}} \times \underset{}{\cancel{100}}} = 6.3548 \times 142. = \$90.24.$$

Explanation.

1. $4\% = \frac{4}{100}$. 3 yr. 6 mo. 18 da. = 42.6 mo.

2. The principal multiplied by the rate, $\frac{4}{100}$, gives the interest for 1 yr.; the interest for 1 yr., divided by 12, gives the interest for 1 mo.; the interest for 1 mo. multiplied by the number of months gives the interest for full time. By cancellation we find \$90.24 to be the required interest.

5. Find the interest of \$480 for 2 yr. 11 mo. 10 da. at 5%.

Process.

$$\frac{\cancel{24}\ \ 212}{\cancel{480} \times \cancel{5} \times \cancel{1060}} = \frac{212}{3} = \$70.67.$$
$$\cancel{100} \times \cancel{360}\ \cancel{15}$$
$$\cancel{20}\ \ 3$$

Explanation.

1. $5\% = \frac{5}{100}$. 2 yr. 11 mo. 10 da. = 1060 da.

2. Finding the interest for one year, as before, we then divide the interest for one year by 360, and thus get the interest for 1 day. The interest for 1 da. multiplied by the number of days gives the interest for full time.

The Principal + the Int. = the Amount. \$480 + \$70.67 = \$550.67.

6. Find the interest and the amount of:

1. \$470.35 for 3 yr. 8 mo. 18 da. at 6%.
2. \$516.50 for 4 yr. 6 mo. 15 da. at 7%.
3. \$318.47 for 3 yr. 7 mo. 10 da. at 5%.
4. \$534.46 for 4 yr. 9 mo. 27 da. at 7%.
5. \$830.27 for 4 yr. 8 mo. 17 da. at 6%.
6. \$579.47 for 4 yr. 10 mo. 12 da. at 5%.
7. \$320.58 for 3 yr. 6 mo. 13 da. at 7%.
8. \$436.45 for 2 yr. 2 mo. 24 da. at 8%.
9. \$547.44 for 5 yr. 3 mo. 19 da. at 4%.
10. \$308.56 for 6 yr. 4 mo. 9 da. at $3\frac{1}{2}$%.

GENERAL REVIEW.

1. Add:

1.	2.	3.
\$295,746.97½	\$374,116.96	\$6,604.00
36,905.73	49,573.88	56,948.35
9,867.96	260,087.61	35,439.50
999.68	60,429.03	678,950.70
5,437.99¼	47,596.84	30,597.05
69,141.16	10,970.55	65,300.40
309,609.78	87,046.00	3,689.74
48,765.99	9,900.67	439.00

2. Find the sum of .37, 6.3, .009, $10.7\dot{4}$, 1.07, 58.93, 748.57 and 2.034.

3. Find the value of $9\frac{999}{1000} + 9\frac{8}{10} + 608 + 39\frac{51}{100} + \frac{7}{20} + 3.1416 + 231$.

4. Add 25 days 7 hours 51 minutes and 26 days 28 hours and 21 minutes.

5. Find the value of 35 bu. 3 pk. + 35 bu. 3 pk. + 35 bu. 3 pk.

6. Add 24 lb. 7 oz., 6 lb. 10 oz., 36 lb. 11 oz., 5 lb. 8 oz.

7. Add 12 rd. 4 yd. 2 ft., 5 rd. 2 ft., 5 yd. 1 ft., 7 rd.

8. Find the sum of $60\frac{4}{25} + 49\frac{3}{50} + 18\frac{7}{20} + 6\frac{3}{4} + 90\frac{1}{2}$.

9. Find the sum of $\frac{5}{8} + \frac{5}{7} + \frac{5}{6} + 5\frac{3}{7}$.

10. Add 234 cu. yd. 18 cu. ft. 566 cu. in., 149 cu. yd. 9 cu. ft. 19 cu. in., 198 cu. yd. 11 cu. ft. 1000 cu. in., 70 cu. yd. 23 cu. ft. 1267 cu. in.

	(1.)	(2.)	(3.)
11.	From 9932563	From 6875396	From 8735009
	take 7953240	take 5927387	take 7295394

12. Find the value of:

(1.)	(2.)	(3.)	(4.)
$3\frac{1}{4} - \frac{2}{3}$.	$17\frac{2}{5} - 11\frac{3}{4}$.	$5\frac{5}{6} - 1\frac{7}{8}$.	$18\frac{3}{5} - 11\frac{5}{7}$.

(5.)	(6.)
7320.00 — 6837.13.	4197.23 — 2076.88.

(7.)	(8.)
173.09 — 134.137.	8394.46 — 4153.76.

13. From five hundred eighty *and* sixty-seven ten-thousandths take ninety-six *and* forty-nine millionths.

14. George Washington was born Feb. 22, 1732. How old was he July 4, 1776?

15. From 16 rd. take 4 rd. 1 yd. 1 ft.

16. From 7 bu. 3 pk. 6 qt. take 6 bu. 2 pk. 7 qt.

17. How much is left if you take 4 gal. 3 qt. 1 pt. from a milk-can containing 10 gal.?

18. Subtract 59.9078 from 64.08.

19. Find the value of (28 — 9) — (27 — 25)?

20. A man bought a factory for $27,000, and sold it for $25,310.625. How much did he lose?

---

21. Multiply: 39,086 by 3049; 27,389 by 8375; 43,009 by 3468.

22. Find the value of $5\frac{1}{2} \times 7\frac{1}{3}$; $16\frac{3}{19} \times 1\frac{7}{9}$; $7\frac{3}{10} \times 9\frac{1}{8}$; $3.1\frac{1}{4} \times 2\frac{5}{9\frac{1}{2}}$.

23. Find the value of: .07646 × 76; 10.025 × 7.29; .036 × .94$\frac{1}{3}$.

24. What cost 87 yd. of muslin at 16$\frac{2}{3}$ cents per yard?

25. What is the value of 2758 bu. of wheat at $.70$\frac{1}{2}$ per bushel?

26. Multiply 4 mi. 130 rd. 11 ft. by 38.

27. What is the value of .875 A. of land at $.33$\frac{1}{3}$ per square rod?

28. Find the value of $1\frac{1}{2} \times \frac{7}{10}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{1\frac{3}{5}}{9}$.

29. Find the value of ($\frac{3}{5}$ of $\frac{7}{8}$) + ($\frac{2}{3}$ of $\frac{6}{7}$) — ($\frac{4}{5}$ of 2).

30. Multiply 10 hhd. 20 gal. 3 pt. by 9.

---

31. Divide 62,098,347 by 5009; 87,329,046 by 8920.

32. Divide $\frac{7}{10}$ by $\frac{3}{5}$; $17\frac{16}{21}$ by $\frac{11}{12}$; $7\frac{3}{8}$ by $3\frac{3}{10}$; 7 by $12\frac{2}{5}$.

33. Divide .504 by .024; 123.6 by .01; 829.31 by .019.

34. Divide 26 mi. 28 rd. $14\frac{1}{2}$ ft. by 7; 74 cd. 19 cu. ft. by 9.

35. Find the value of $2\frac{1}{2} \times (\frac{3}{4} \div \frac{2}{3}) \times \frac{1}{3}$.

36. Multiply $3\frac{2}{13}$ by $17\frac{5}{7}$, and divide the result by $2\frac{1}{2}$ of $2\frac{2}{7}$.

37. If a barrel of flour is worth $\$6\frac{3}{8}$, how many barrels will \$510 buy?

38. If 24 men do a piece of work in 105 days, how long will it take 72 men to do it?

39. At $18\frac{3}{4}$ cents per C., how many laths can be bought for \$37.50.

40. Divide $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of $\frac{5}{9}$ by $\frac{5}{6}$ of $\frac{8}{9}$ of $\frac{3}{10}$ of 4.

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41. Find by cancellation the value of $\frac{111 \times 5 \times 28 \times 1}{37 \times 7 \times 21 \times 50}$; $\frac{144 \times 17 \times 45 \times 52}{13 \times 9 \times 34 \times 12}$; $\frac{57 \times 119 \times 16}{17 \times 12 \times 19}$.

42. How many dresses, each containing 15 yd., can be made from 25 pieces of cloth, each containing 45 yd.?

43. Reduce to lowest terms: $\frac{35}{75}$, $\frac{3249}{4500}$, $\frac{4751}{5044}$, $\frac{1617}{2156}$.

44. Find the prime factors of 295, 556, 648, 1063, 1495.

45. Reduce to higher terms: $\frac{5}{9}$ to 27ths, $\frac{29}{63}$ to 819ths, $\frac{7}{10}$ to 1000ths.

46. Reduce to whole or mixed numbers: $\frac{699}{21}$, $\frac{3611}{44}$, $\frac{1875}{30}$, $\frac{1898}{144}$, $\frac{1776}{4}$.

47. Reduce to improper fractions: $68\frac{3}{7}$, $400\frac{2}{91}$, $96\frac{70}{111}$, $3.1\frac{1}{4}$.

48. Reduce 4 bbl. 5 gal. 1 pt. to gills; 5 hhd. $7\frac{1}{2}$ gal. to pints.

49. Reduce to higher denominations: 845,356 in.; 1,000,000 cu. ft.; 7016 dr.

50. Reduce to lower denominations: $\frac{5}{8}$ T.; .75 bu.; .81 sq. mi.; .385 da.

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51. Find the value of $(4\frac{2}{3} + 6\frac{1}{4} - 3\frac{2}{5}) \div \frac{1}{5} + (3\frac{1}{8} - 2\frac{1}{4}) \times 6$.

52. Find the value of 81.8 + 35.625 — 38.875 — 2.0034.

53. What is the sum of 303 thousandths, 4108 millionths, 635 ten-thousandths, 803 ten-millionths?

54. Multiply .8745 by 100; by 1000; by 100,000; by 1,000,000.

55. Divide .8745 by 100; by 1000; by 100,000; by 1,000,000.

56. Divide .08 by 1.611; 40,000 by .00004; 144 by 1728.

57. How many barrels of apples, at $2.25 per barrel, can be bought for $29\frac{1}{4}$?

58. If illuminating gas is sold at the rate of $1.00 per M. cubic feet, how much will 52,437 cu. ft. cost?

59. If a railroad train runs 35.75 miles per hour, in how many hours will it run 143 miles?

60. A farmer sold $21\frac{1}{2}$ dozen eggs at $.18\frac{3}{4}$ a dozen, and bought $14\frac{1}{4}$ yd. of cloth at $12\frac{1}{2}$ cents a yard. How much money had he left?

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61. How many acres in a rectangular field 70 rd. wide and 90 rd. long?

62. The length, breadth, and thickness of a solid are, respectively, 9 ft. 4 in., 10 ft. 6 in., and 7 ft. 8 in. Find its volume.

63. How many cords of wood in a pile 4 rd. long, $2\frac{1}{2}$ yd. high, and 4 ft. wide?

64. How many cubic inches in a cubic rod?

65. How many board feet in 38 planks 10 in. wide, 16 ft. long, and 2 in. thick?

66. How many acres in a rectangle $24\frac{1}{2}$ rd. long by 16.02 rd. wide?

67. How many square feet in the 4 sides of a room $21\frac{1}{2}$ ft. long, $16\frac{1}{2}$ ft. wide, and 13 feet high?

68. What is the cost of a field 173 rd. long and 84 rd. wide at $35.75 per acre?

69. How many board feet in a 3-inch plank 18 ft. long and 14 in. wide?

70. How much will a stick of timber 40 ft. 9 in. long, 1 ft. 3 in. wide, and 1 ft. 9 in. thick cost, at 25 cents a cubic foot?

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71. Find $66\frac{2}{3}\%$ of $314.16.

72. Find the interest of $57.35 for 90 da., at 6%.

73. Find the amount of $691.75 for 15 da., at 6%.

74. A money-lender loaned $840.50 for 2 yr. and 8 mo., at $5\frac{1}{2}\%$. What amount of money was due him at the end of the time?

75. I loaned a friend $460 on Jan. 1, 1898, at 6% interest. If the loan is not repaid till Jan. 1, 1900, how much money will then be due me?

76. Find the amount of $108.46 for 4 yr. 8 mo. 5 da., at 10%.

77. Find the interest and amount of $500 for 2 yr. 2 mo. 2 da., at 6%.

78. Find the interest of $1000 for 30 days. For 33 days.

79. $100, put on interest Dec. 1, 1898, will amount to how much money on Dec. 31, 1899, at 5%, reckoning 365 days in a year?

80. I bought a horse for $90.00. I sold him at a gain of 25%. I loaned the money thus received, for a year, at 7%. How much money was due me at the end of the year?

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81. Two men start at the same time and place and travel in opposite directions, one at the rate of 35 mi. per day, the other at the rate of 42 mi. per day. How far will they be apart at the end of 19 days?

82. A., B. and C. have together $209. A. and C. have $155; A. and B. have $109. How much has each?

83. The greater of two numbers is 5067, and their difference is 4760. What is the less number?

84. The divisor is 645 and the quotient 43. What is the dividend?

85. The subtrahend is 45,304 and the remainder is 9807. What is the minuend?

86. A wagon wheel is 16 ft. 8 in. in circumference. How many revolutions will it make in going 5 miles?

87. If $\frac{3}{5}$ of an acre of land produces 120 bushels of potatoes, how many bushels will $4\frac{1}{4}$ acres produce?

88. A man who owned $\frac{5}{8}$ of a mill sold $\frac{1}{5}$ of his share for $2500. What was the value of the mill at that rate, and what the value of the man's share?

89. James can do a piece of work in 12 days, and

Robert can do it in 10 days. In what time can they both do it?

90. Three children inherited in equal shares a farm of 473 A. 112 sq. rd. How much land did each receive?

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91. A vintner has 168 gal. 3 qt. 1 pt. of wine. How much is it worth at $\$2.62\frac{1}{2}$ per gallon?

92. 4 T. 17 cwt. 75 lb. of hay being sold brought $10.75 per ton. What was the total receipt for the hay?

93. How many dresses containing 8 yd. 2 ft. each can be made out of 390 yd. of cloth?

94. A lot contains 12 A. It is 30 rd. wide. How long is it?

95. A. has $\frac{3}{4}$ as much money as B. They both have $581. How much has each?

96. A room measures on the floor 20 ft. by 18 ft. How much carpet will cover it, and what will the carpet cost at $\$1.12\frac{1}{2}$ per square yard?

97. I paid a debt of $295.85, which had been upon interest 11 mo. 25 da. at 7%. What amount did I pay?

98. What number divided by $1\frac{3}{8}$ will give a quotient of $4\frac{3}{7}$?

99. A merchant bought 1250 barrels of flour for $6250. At what price per barrel must he sell it to make a profit of $12\frac{1}{2}\%$?

1. Percentage = Base × Rate.
2. Interest = Percentage × Time.
3. Interest = Base × Rate × Time.

100. Base = ? [1 and 3.] Rate = ? [1 and 3.] Time = ? [2 and 3.]

ANSWERS.

ADDITION.

Page 70.

2. 1. 24.
2. 22.
3. 24.
4. 19.
5. 28.
6. 20.
7. 21.
8. 21.
9. 24.
10. 24.
11. 23.
12. 25.
13. 27.
14. 15.
15. 23.
16. 17.
17. 22.
18. 23.
19. 17.
20. 21.
21. 28.
22. 28.
23. 27.
24. 24.
25. 25.
26. 25.
27. 35.
28. 33.
29. 28.
30. 28.
31. 35.
32. 21.
33. 26.
34. 30.
35. 23.
36. 24.
37. 29.
38. 27.
39. 29.
40. 28.

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8. 1. 29.
2. 33.
3. 29.
4. 32.
5. 30.
6. 34.
7. 37.
8. 32.
9. 29.
10. 28.
11. 39.
12. 29.
13. 27.
14. 37.
15. 31.
16. 28.
17. 30.
18. 41.
19. 31.
20. 36.

1. 18.
2. 12 years.
3. 24 feet.
4. 33 books.

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5. 15 pieces.
6. 13 gills.
7. 11 pounds.
8. 18 problems.
9. 26 feet.
10. 40 inches.
11. 17 boys; 11 girls; 28 pupils.
12. 16 units; 36 cents.

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13. 33 lines. 14. 45. 15. 35.

Page 74.

2. 1. 1368.
2. 1125.
3. 1531.
4. 1760.
5. 1652.
6. 688.
7. 492.
8. 589.

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9. $708.
10. $38.23.
11. $225.44.
12. $137.44.
13. $175.81.
14. $2874.42.

3. 1. 90309.
2. 10970.
3. 9055.
4. 5293.

4. 26765.
5. 13593.
6. 22070.
7. 8110.
8. 135929.
9. 243076.
10. $278.85.
11. $3261.69.
12. $1398.93.

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13. $342.809.
14. $833.783.
15. $710.688.

16. 1. 56065.
2. 15071.
3. 24004.
4. 435000.
5. 5004867.
6. 547000000.
7. 29000788.
8. 29900.
9. 9006439.
10. 21847.

17. 1. $1160.83. 3. $703.78.
2. $382.99.

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4. $1688.02. 5. $1500.3725.

1. $16992. 3. $800. 5. $8818.
2. 47624. 4. 21.00. 6. 1164.

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7. $126.90; 217.
8. 2,622,254.
9. $1796.
10. $9265.
11. 842.
12. 16906.
13. $8360.
14. 817; $8.17; 125.

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15. 1. 13701.
2. 12248.
3. 135667.
4. 128257.
5. $\overline{\text{I}}\text{X}$DCCCXCVIII = 9898.

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2. 1. 211. 8. 341. 15. 1343.
2. 552. 9. 210. 16. 4200.
3. 341. 10. 400. 17. 3213.
4. 430. 11. 351. 18. 3541.
5. 502. 12. 532. 19. 3233.
6. 201. 13. 440. 20. 7111.
7. 633. 14. 311.

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21. $22.11. 27. 61.12.
22. 24.20. 28. 71.000.
23. 16.13. 29. 45.90.
24. 13.45. 30. 10.426.
25. 36.14. 31. 610.17.
26. 53.35.

1. 702. 4. 3420. 7. 101.20.
2. 1.00. 5. ——. 8. 252.15.
3. 2.50. 6. ——. 9. 3131.

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10. 1000. 15. 1732.
11. 2030. 16. 401.
12. 5504. 17. ——.
13. 1115. 18. 202.20.
14. 3632. 19. 106.

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3. 1. 169. 16. 76.
2. 212. 17. 20.39.
3. 391. 18. 7.92.
4. 329. 19. 16.48.
5. 264. 20. 21.05.
6. 387. 21. 9.25.
7. 115. 22. 35.38.
8. 32. 23. 35.91.
9. 66. 24. 28.41.
10. 509. 25. 449.48.
11. 82. 26. 269.89.
12. 108. 27. 158.07.
13. 46. 28. 688.89.
14. 567. 29. 2578.99.
15. 167.

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1. 943. 8. 1096.
2. 2031. 9. 1.
3. 1435. 10. 999000.
4. 1206. 11. 593.
5. 35433. 12. 125.77.
6. 1090. 13. 146.882.
7. 77655. 14. 92760000.

5. 1. 391.
2. 417.
3. 387.
4. 51.
5. 1991.
6. 4886.
7. 1603.
8. 1962.
9. 2218.
10. 54029.
11. 34371.
12. 540100.
13. 459890.
14. 5170174.
15. 136353.
16. 1228608.
17. 132187.
18. 4745334.
19. 307139.
20. 1979323.

1. 7202.
2. 951691.
8. 52581.

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4. 6462.
5. 11020.
6. 6757.
7. 332100.
8. 999891.
9. 976000000.
10. 298,470,000.
11. 10,179,000; 19642000.
12. $204.96.
13. 46.8.
14. 1128.

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1. $7.
2. 10.
3. 9.
4. 15.
5. 17.
6. $12.
7. 65.
8. $2.
9. $19.50.
10. 1801.

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11. 0.
12. 18.
13. 28.
14. $80.
15. $3.00.

2. 1. 10654.
2. 2185.
3. 2367.
4. 50895.
5. 11388.
6. 885.
7. 30443.
8. 58852.
9. 2517.
10. 487783.

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3. 7202.
4. $14750.
5. $16675.
6. $2693.23.
7. 33064.
8. $16.75.
9. 94.
10. $8454.20.
11. $2.10.
12. $22.735.

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13. A. 450; B. 625; C. 961.
14. 0.
15. $11251.
16. 1578620818.
17. 87.
18. $1056.75.
19. 217.
20. Summer, 2 days.
21. $87.28.

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22. Mary, 5 cts.
23. $3344; $4162.84.
24. 24; 34; 6.684.
25. $.50; $.375; $.625.
26. 936085.
27. 1216738.
28. $3.00.
29. DCXXXI.
30. C.
31. $9850.
32. $7240; $2760.

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1. 1735.
2. 2192.
3. 2120.
4. 3748.
5. 1740.
6. 2715.
7. 3420.
8. 4715.
9. 12288.
10. 41559.
11. 53670.
12. 42266.
13. 71856.
14. 50768.
15. 48564.
16. 21980.
17. 482805.
18. 184344.
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20. 376096.
21. 395586.
22. 253664.
23. 340784.
24. 213409.
25. 246912.
26. 2367036.
27. 1382712.
28. 4506170.
29. 3407340.
30. 864192.
31. 7120984.
32. 4111101.
33. 4382715.

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1. 31.50.
2. 200.
3. 2.50.
4. 864.
5. 2920.
6. 10080.
7. 15052.94.
8. 31680.

9. 1. 56.
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   3. 62.
   4. 88.
   5. 55.
   6. 16.
   7. 98.
   8. 88.
   9. 54.
   10. 0.

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    5. 1810659.
    6. 1439615.
    7. 4150827.

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5. 1. 57900.
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   6. 450000.
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   8. 5700000.
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   10. 14850.

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11. 22800.
12. 23100.
13. 54320.
14. 182100.
15. 203200.
16. 543200.
17. 171600.
18. 653800.
19. 3072000.
20. 4690000.
21. 227200.
22. 46100.
23. 9876000.
24. 93520.
25. 24048000.
26. 3288000.
27. 7952000.
28. 4736000.
29. 10728000.
30. 994000.

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8. 1. 15910.
   2. 20860.
   3. 31680.
   4. 45144.
   5. 59031.
   6. 43148.
   7. 32795.
   8. 61050.
   9. 39220.
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   13. 620412.
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   24. 1914.84.
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   60. 70109427840.

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1. 85064.
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4. 1470.
5. 110048.
6. 31710.

7. 4379.48.
8. 8565.78.
9. 89760.
10. 8883.
11. 651222.
12. 614680.
13. 2243160.
14. $12000.

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15. $878.85.

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1. 24791.
2. 175.

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3. 600.
4. 3125.20.
5. 75881.
6. 720.
7. $2140920.
8. 7883.
9. $72035.
10. 424.
11. 999000.
12. 99900000.
13. 5820.
14. 196.80.
15. 35.
16. 1440.

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17. 1806.
18. 58.24.
19. 12960.
20. 374.
21. 5219.
22. 636 profit.
23. 12 profit.
24. 2400.
25. 1. 600.

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2. 698.
3. 79962.
4. $1798.98.

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2. 1. 321.
2. 321.
3. 918.
4. 541.
5. 307.
6. 647.
7. 315.
8. 337.
9. 384.
10. 493.
11. 1225.
12. 1084.
13. 1218.
14. 1366.
15. 496.
16. 597.
17. 1545.
18. 984.
19. 406.
20. 876.
21. $5.54.
22. $13.53.
23. $5.48.
24. $12.34.
25. $9.12.
26. $13.89.
27. $9.08.
28. $22.96.
29. $8.88.
30. $3.91.

	Q.	R.		Q.	R.
4. 1.	368	1.	15.	1228	6.
2.	245	1.	16.	1406	5.
3.	240	3.	17.	1058	2.
4.	183	1.	18.	985	3.
5.	137	2.	19.	24371	1.
6.	133	6.	20.	17914	0.
7.	124	3.	21.	16472	1.
8.	107	2.	22.	9479	2.
9.	84	7.	23.	9618	1.
10.	675	5.	24.	8256	2.
11.	1220	7.	25.	13367	1.
12.	1979	1.	26.	12084	2.
13.	1099	3.	27.	6387	4.
14.	3189	1.			

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	Q.	R.		Q.	R.
28.	197	1.	45.	1035	8.
29.	154	3.	46.	1587	2.
30.	137	6.	47.	1624	2.
31.	248	3.	48.	1158	4.
32.	71	6.	49.	245	4.
33.	382	1.	50.	12865	1.
34.	124	6.	51.	19739	1.
35.	192	3.	52.	13796	3.
36.	107	2.	53.	11837	5.
37.	239	1.	54.	7765	1.
38.	162	2.	55.	28780	0.
39.	532	4.	56.	22669	2.
40.	529	4.	57.	16560	5.
41.	1206	3.	58.	31240	0.
42.	945	2.	59.	11853	7.
43.	1247	2.	60.	10660	2.
44.	1404	4.			

1. 200.
2. 176.
3. 285.
4. 21120.
5. 65654.
6. 62256.
7. 56365

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8. $5\frac{1}{4}$.
9. 162; 126.
10. 588.
11. 144 hr.
12. 4688.
13. $7351.
14. 1120.
15. 4840.
16. 52.
17. 6542.
18. 248.
19. 1342.
20. 180.

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4. 1. $8\frac{9}{10}$.
2. $8\frac{78}{100}$.
3. $34\frac{22}{100}$.
4. $5\frac{489}{1000}$.
5. $6\frac{72}{1000}$.
6. $715\frac{60}{100}$.
7. $970\frac{48}{100}$.
8. $57\frac{84}{1000}$.
9. $97\frac{68}{1000}$.
10. $95\frac{650}{1000}$.
11. $198\frac{1}{40}$.
12. $61\frac{23}{50}$.
13. $16\frac{147}{200}$.
14. $10\frac{97}{500}$.
15. $45\frac{798}{2000}$.
16. $32\frac{704}{3000}$.
17. $18\frac{2970}{4000}$.
18. $94\frac{885}{3000}$.
19. $12\frac{2762}{5000}$.
20. $11\frac{5544}{7000}$.
21. $12\frac{8882}{8000}$.
22. $11\frac{953}{9000}$.
23. $123\frac{456}{1000}$.
24. $394\frac{1012}{2000}$.
25. $115\frac{675}{3000}$.
26. $225\frac{1235}{4000}$.
27. $135\frac{1012}{1600}$.
28. $57\frac{2478}{3000}$.
29. $28\frac{2784}{7000}$.
30. $70\frac{2801}{8000}$.

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5. 1. 789.
2. $482\frac{1}{11}$.
3. $305\frac{7}{12}$.
4. $583\frac{11}{13}$.
5. $292\frac{1}{14}$.
6. $605\frac{10}{15}$.
7. $154\frac{12}{16}$.
8. $201\frac{8}{17}$.
9. $144\frac{7}{18}$.
10. $203\frac{1}{19}$.
11. 221.
12. $219\frac{6}{21}$.
13. 209.
14. $135\frac{1}{14}$.
15. $135\frac{10}{15}$.
16. $81\frac{12}{56}$.
17. $66\frac{58}{87}$.
18. $53\frac{1}{78}$.
19. $41\frac{52}{85}$.
20. $74\frac{8}{95}$.
21. $112\frac{2}{78}$.
22. $53\frac{28}{63}$.
23. $83\frac{34}{42}$.
24. $73\frac{15}{77}$.
25. $128\frac{28}{56}$.
26. 221.
27. $219\frac{1}{42}$.
28. 209.
29. $135\frac{24}{63}$.
30. $159\frac{47}{57}$.
31. 1052.
32. $994\frac{5}{16}$.
33. $862\frac{14}{17}$.
34. $2108\frac{6}{18}$.
35. $1291\frac{5}{19}$.
36. $3179\frac{15}{20}$.
37. $577\frac{22}{30}$.
38. $547\frac{10}{50}$.
39. $346\frac{32}{60}$.
40. $495\frac{3}{70}$.
41. $552\frac{40}{80}$.
42. $511\frac{60}{90}$.
43. $144\frac{21}{100}$.
44. $136\frac{70}{101}$.
45. $239\frac{3}{103}$.
46. $188\frac{27}{121}$.
47. $218\frac{54}{123}$.
48. $222\frac{85}{130}$.
49. 206.
50. $423\frac{18}{156}$.
51. $511\frac{20}{160}$.
52. $316\frac{120}{172}$.
53. $448\frac{128}{185}$.
54. $27\frac{65}{190}$.
55. $88\frac{114}{205}$.
56. $86\frac{50}{205}$.
57. $125\frac{5}{231}$.
58. $139\frac{92}{276}$.
59. $158\frac{200}{300}$.
60. $245\frac{28}{294}$.

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6. 1. $917\frac{208}{324}$.
2. $1452\frac{27}{404}$.
3. $450\frac{128}{528}$.
4. $389\frac{140}{783}$.
5. $1375\frac{178}{250}$.
6. $463\frac{51}{684}$.
7. $301\frac{457}{674}$.
8. $397\frac{89}{879}$.
9. $839\frac{78}{709}$.
10. $860\frac{425}{534}$.
11. $859\frac{72}{378}$.
12. $819\frac{282}{402}$.
13. $913\frac{110}{723}$.
14. $819\frac{705}{3245}$.
15. $669\frac{456}{5023}$.
16. $668\frac{1188}{6438}$.
17. $275\frac{4781}{7253}$.
18. $574\frac{8888}{8007}$.
19. $423\frac{4701}{7035}$.
20. $636\frac{4221}{8294}$.
21. $246\frac{8772}{7272}$.
22. $1594\frac{7781}{5955}$.
23. $1001\frac{2888}{7988}$.
24. $262\frac{978}{5757}$.
25. $1080\frac{5888}{8888}$.
26. $921\frac{2050}{4949}$.
27. $1047\frac{5488}{6787}$.
28. $719\frac{4417}{8898}$.
29. $935\frac{2296}{4864}$.
30. $679\frac{4488}{7373}$.
31. $1301\frac{4895}{8989}$.
32. $452\frac{2295}{3883}$.

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1. 115.
2. 25.
3. 135.
4. 135.
5. $60\frac{7}{107}$.
6. 16.

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7. 14. 12. 539. 17. 280.
8. 4732. 13. 135. 18. 25.
9. 2093. 14. 21890. 19. 2.
10. 130. 15. 329.
11. $494\frac{110}{185}$. 16. 480.

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20. 75. 22. $3\frac{115}{360}$. 24. 86.
21. $16\frac{8125}{1000}$. 23. 97.

1. 228. 4. 147. 7. 180.
2. 49. 5. 183.
3. 120. 6. 20.

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8. 143. 11. 130. 14. 273.
9. 405. 12. 5. 15. 78.
10. 60. 13. 26. 16. 3.151.

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4. 61600. 9. 15. 14. 1632.
5. 360. 10. 40. 15. 198.
6. 1568. 11. 6480. 16. 17.
7. 246. 12. 93.
8. 946. 13. 148.

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17. 119. 22. 15. 27. 118.
18. 17052. 23. 29. 28. 4.80.
19. 95. 24. 17. 29. 1200.
20. 241.92. 25. 5.
21. 33. 26. 33.67.

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30. 10. 34. 24. 38. 17.
31. $110. 35. 5684. 39. 840.
32. 2552. 36. 18. 40. 163504.
33. 9.75. 37. 50.

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41. 784. 43. 20.
42. 138. 44. XLIV.

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2. 1. 2, 2, 3, 19.
2. 2, 2, 3, 3, 3, 3.
3. 2, 2, 2, 2, 2, 7.
4. 2, 2, 2, 43.
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7. 2, 2, 199.
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24. 2, 2, 5, 5, 5.
25. 5, 193.
26. Has no factors.
27. 11, 83.
28. 7, 149.
29. 2, 2, 2, 3, 5, 7.
30. 2, 3, 3, 5, 11.
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40. 2, 2, 2, 3, 7, 11, 13.

41. 2, 2, 2, 2, 2, 2, 3, 5, 5.
42. 17, 29.
43. 17, 41.
44. 3, 3, 1213.

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3. 1. $2\frac{3}{35}$. 2. 20. 3. $2\frac{1}{2}$. 4. $1\frac{23}{35}$. 5. 4. 6. 16. 7. 36. 8. 47. 9. 1. 10. 3. 11. 1. 12. $7\frac{1}{2}$. 13. 4. 14. 35. 15. $1\frac{1}{7}$. 16. 1. 17. 11. 18. $6\frac{12}{25}$. 19. 8. 20. $2\frac{1}{2}$. 21. 2. 22. 32. 23. 14. 24. 20. 25. $2\frac{1}{3}$.

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2. 44. 3. 60. 4. 80. 5. 10. 6. $4\frac{1}{2}$. 7. $37\frac{1}{2}$. 8. 15. 9. 60. 10. 3000. 11. 40.

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12. 36. 13. 3072. 14. 24. 15. 823.5. 16. 30. 17. 55.

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2. 1. $\frac{1}{2}$. 2. $\frac{1}{2}$. 3. $\frac{1}{2}$. 4. $\frac{1}{3}$. 5. $\frac{1}{3}$. 6. $\frac{2}{3}$. 7. $\frac{5}{18}$. 8. $\frac{3}{5}$. 9. $\frac{1}{4}$. 10. $\frac{5}{6}$. 11. $\frac{7}{9}$. 12. $\frac{3}{5}$. 13. $\frac{4}{5}$. 14. $\frac{2}{9}$. 15. $\frac{4}{9}$. 16. $\frac{7}{18}$. 17. $\frac{5}{8}$. 18. $\frac{5}{7}$. 19. $\frac{4}{15}$. 20. $\frac{1}{4}$. 21. $\frac{16}{17}$. 22. $\frac{24}{25}$. 23. $\frac{12}{13}$. 24. $\frac{2}{3}$. 25. $\frac{3}{4}$. 26. $\frac{9}{16}$. 27. $\frac{1}{11}$. 28. $\frac{7}{9}$. 29. $\frac{1}{3}$. 30. $\frac{5}{7}$. 31. $\frac{7}{8}$. 32. $\frac{7}{8}$. 33. $\frac{3}{4}$. 34. $\frac{5}{11}$. 35. $\frac{4}{11}$. 36. $\frac{6}{7}$. 37. $\frac{10}{11}$. 38. $\frac{5}{8}$. 39. $\frac{9}{40}$. 40. $\frac{1}{12}$. 41. $\frac{53}{176}$. 42. $\frac{7}{8}$. 43. $\frac{1}{3}$. 44. $\frac{9387}{15555}$. 45. $\frac{21}{29}$. 46. $\frac{5}{18}$. 47. $\frac{64}{125}$. 48. $\frac{5}{21}$. 49. $\frac{1}{2}$. 50. $\frac{1}{30}$.

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3. 1. 168. 2. 56. 3. 120. 4. 8. 5. 30. 6. 48. 7. 20. 8. 28. 9. 36. 10. 44. 11. 24. 12. 100. 13. 40. 14. 36. 15. 104. 16. 840. 17. 120. 18. 540. 19. 315. 20. 504. 21. 72. 22. 60. 23. 420. 24. 12600. 25. 192. 26. 360. 27. 924. 28. 357. 29. 150. 30. 336. 31. 252. 32. 616.

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4. 1. 27. 2. $12\frac{1}{12}$. 3. $35\frac{1}{24}$. 4. $113\frac{143}{180}$. 5. $82\frac{3}{44}$. 6. $112\frac{1}{2}$. 7. $7\frac{16}{101}$. 8. $5\frac{68}{143}$. 9. $1\frac{141}{340}$. 10. $62\frac{1}{2}$. 11. $8\frac{98}{135}$. 12. $3\frac{387}{380}$. 13. $6\frac{1}{20}$. 14. $164\frac{1}{2}$. 15. $80\frac{18}{100}$. 16. $5\frac{144}{343}$.

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2. 1. $\frac{139}{8}$. 2. $\frac{4885}{12}$. 3. $\frac{2148}{47}$. 4. $\frac{1139}{16}$. 5. $\frac{1467}{17}$. 6. $\frac{1711}{100}$. 7. $\frac{2953}{100}$. 8. $\frac{23005}{246}$. 9. $\frac{1092}{33}$. 10. $\frac{5340}{31}$. 11. $\frac{17822}{35}$. 12. $\frac{7183}{23}$. 13. $\frac{16259}{29}$. 14. $\frac{84025}{337}$. 15. $\frac{98034}{75}$. 16. $\frac{46255}{31}$.

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2. 1. $2\frac{3}{5}$. 2. $1\frac{23}{126}$. 3. $1\frac{31}{60}$. 4. $1\frac{9}{280}$. 5. $2\frac{1}{10}$. 6. $1\frac{19}{60}$. 7. $2\frac{5}{24}$. 8. $5\frac{3}{4}$. 9. $4\frac{3}{28}$. 10. $2\frac{3}{10}$. 11. $1\frac{43}{140}$. 12. $1\frac{9}{10}$. 13. $10\frac{1}{12}$. 14. $12\frac{178}{315}$. 15. $1\frac{27}{44}$. 16. $14\frac{86}{105}$. 17. $7\frac{113}{132}$. 18. $10\frac{43}{56}$. 19. $19\frac{7}{120}$. 20. $20\frac{47}{48}$. 21. $1\frac{68}{121}$.

Page 159.

5. 1. $2\frac{4}{3}\frac{2}{5}$.
2. $23\frac{7}{1}$.
3. $21\frac{1}{3}\frac{1}{0}$.
4. $18\frac{7}{2}$.
5. $12\frac{8}{1}\frac{3}{6}$.
6. $90\frac{1}{3}\frac{7}{0}$.
7. $364\frac{1}{1}\frac{5}{6}$.
8. $121\frac{3}{8}$.
9. $48\frac{5}{8}$.
10. $35\frac{1}{2}$.
11. $47\frac{1}{1}\frac{1}{0}$.
12. $\frac{47}{100}$.
13. $74\frac{1}{1}\frac{1}{0}$.
14. $23\frac{7}{1}\frac{8}{2}\frac{5}{5}$.
15. $146\frac{1}{4}\frac{7}{8}$.
16. $\frac{1}{1}\frac{2}{6}\frac{5}{6}$.
17. $57\frac{1}{2}$.
18. $17\frac{2}{4}\frac{9}{0}$.
19. $4\frac{1}{3}\frac{4}{9}\frac{9}{6}\frac{2}{0}\frac{1}{0}$.
20. $10\frac{2}{3}\frac{0}{9}\frac{2}{6}\frac{1}{0}$.

1. $34\frac{3}{4}$.
2. $75\frac{8}{4}\frac{3}{2}\frac{8}{0}$.

Page 160.

3. $144\frac{3}{1}\frac{6}{0}\frac{7}{4}$.
4. $1.56\frac{1}{8}$.
5. 12.
6. $18\frac{4}{2}\frac{1}{5}$.
7. $123\frac{2}{8}\frac{4}{0}\frac{3}{0}$.
8. $44\frac{1}{4}\frac{2}{0}$.
9. 31.
10. $141\frac{7}{3}\frac{0}{0}$.
11. $136\frac{9}{1}\frac{3}{0}\frac{8}{2}\frac{4}{3}$.
12. $162\frac{9}{1}\frac{9}{3}\frac{9}{8}$.
13. $1877\frac{1}{8}$; $666\frac{1}{16}\frac{5}{6}$.
14. $21\frac{3}{3}\frac{1}{6}$.

Page 163.

4. 1. $\frac{2}{3}$.
2. $4\frac{1}{6}$.
3. $\frac{2}{6}\frac{2}{3}$.
4. $\frac{4}{7}\frac{7}{2}$.
5. $\frac{1}{6}$.
6. $\frac{5}{3}\frac{5}{8}$.
7. $1\frac{2}{9}$.
8. $\frac{1}{3}$.
9. $\frac{3}{7}\frac{3}{7}$.
10. $\frac{2}{3}\frac{4}{5}$.
11. $\frac{1}{1}\frac{3}{2}$.
12. $\frac{1}{3}$.
13. $3\frac{3}{5}$.
14. $3\frac{1}{8}$.
15. $7\frac{7}{1}\frac{7}{8}$.
16. $\frac{2}{3}\frac{7}{5}\frac{7}{0}$.
17. $\frac{9}{1}\frac{9}{4}\frac{9}{0}$.
18. $\frac{8}{1}\frac{3}{6}\frac{3}{8}$.
19. $12\frac{5}{6}\frac{5}{3}$.
20. $27\frac{1}{1}\frac{1}{8}$.
21. $17\frac{2}{3}\frac{4}{5}$.
22. $4\frac{6}{3}\frac{6}{8}$.
23. $2\frac{2}{4}\frac{2}{8}$.
24. $8\frac{3}{4}$.
25. $6\frac{4}{5}$.
26. $18\frac{7}{1}\frac{7}{3}$.
27. $5\frac{5}{3}\frac{5}{3}$.
28. $\frac{7}{1}\frac{7}{2}$.
29. $\frac{1}{2}\frac{0}{1}$.
30. $\frac{3}{5}\frac{1}{8}$.
31. $\frac{1}{6}\frac{9}{0}$.
32. $\frac{4}{7}\frac{1}{0}$.
33. $\frac{1}{3}\frac{7}{6}$.
34. $\frac{2}{4}\frac{1}{0}$.
35. $1\frac{7}{1}\frac{7}{5}$.
36. $1\frac{2}{2}\frac{3}{4}$.
37. $\frac{7}{8}$.
38. $8\frac{3}{8}$.
39. $\frac{1}{1}\frac{9}{9}\frac{9}{5}$.
40. $9\frac{2}{3}\frac{2}{7}$.
41. $17\frac{1}{4}\frac{3}{8}$.
42. $3\frac{1}{7}$.

Page 164.

1. $5\frac{1}{8}$.
2. $28\frac{1}{2}\frac{7}{0}$.
3. $41\frac{3}{4}$.
4. $87\frac{7}{1}\frac{8}{3}\frac{8}{3}$.
5. $\frac{11}{15}$; $\frac{4}{15}$.
6. $\frac{1}{2}\frac{9}{0}$.
7. $4\frac{9}{1}\frac{7}{1}\frac{7}{3}$.
8. $\frac{2}{3}\frac{6}{1}\frac{1}{5}$.
9. $\frac{1}{2}\frac{2}{7}\frac{4}{3}$.
10. $9\frac{7}{8}$.
11. $\frac{9}{80}$.
12. $4\frac{1}{2}\frac{9}{1}$.

Page 167.

1. $3.00.
2. $7.50.
3. 14.
4. $12\frac{1}{4}$.
5. $84.
6. $20\frac{4}{5}$ cts.
7. $50\frac{1}{4}$.
8. $134.
9. $117\frac{5}{7}$.

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10. $2338\frac{7}{8}$.
11. $110\frac{1}{2}$.
12. $108\frac{1}{2}$.
13. $409\frac{1}{2}$.
14. $7414.
15. $153.
16. $80\frac{7}{8}$.

Page 170.

4. 1. 10.
2. 21.
3. 140.
4. $\frac{1}{2}$.
5. 56.
6. 228.
7. 258.
8. 412.
9. 462.

5. 1. $\frac{3}{4}\frac{5}{5}$.
2. $3\frac{4}{5}$.
3. $\frac{8}{1}\frac{1}{1}$.
4. $10\frac{4}{5}\frac{7}{5}$.
5. 384.
6. $1207\frac{1}{5}$.
7. 21.
8. $994\frac{2}{7}$.
9. 4275.
10. $3241\frac{7}{8}$.
11. $\frac{8}{1}\frac{5}{5}$.
12. $\frac{3}{5}$.
13. $\frac{7}{1}\frac{0}{0}$.
14. $\frac{8}{2}\frac{5}{5}$.
15. $57\frac{2}{5}$.
16. $41\frac{7}{8}$.
17. $2\frac{2}{3}$.
18. $\frac{5}{1}\frac{5}{5}\frac{5}{3}$.
19. $\frac{7}{2}\frac{7}{0}$.
20. $\frac{1}{2}\frac{8}{5}$.
21. $\frac{2}{5}$.
22. $\frac{7}{3}\frac{7}{0}$.
23. $3\frac{3}{4}$.
24. $5\frac{3}{5}$.
25. $1\frac{1}{3}\frac{2}{1}\frac{9}{1}$.
26. $1\frac{4}{5}$.
27. $7\frac{1}{2}\frac{3}{1}$.
28. $111\frac{1}{3}$.
29. $16\frac{4}{1}\frac{4}{1}$.
30. $3\frac{3}{4}$.

6. 1. $\$11\frac{5}{3}\frac{5}{2}$.
2. $156\frac{3}{5}$.
3. 46.

Page 171.

4. $35\frac{7}{1}\frac{7}{6}$.
5. $44\frac{4}{5}$.
6. $32\frac{3}{8}$.
7. $2.19\frac{2}{3}\frac{7}{8}$.
8. $2.01\frac{9}{1}\frac{9}{6}$.
9. $10\frac{1}{2}$.
10. $3.20\frac{5}{8}$.

1. $8\frac{7}{1}\frac{7}{6}$.
2. $12\frac{1}{4}$.
3. $130\frac{1}{2}$.
4. $232\frac{1}{2}$.
5. $5\frac{4}{5}$.
6. $1336\frac{2}{3}$.
7. $77.
8. $125\frac{1}{1}\frac{1}{4}$.
9. $2\frac{7}{2}\frac{7}{4}$.
10. 140; 40.
11. $41\frac{4}{1}\frac{1}{2}\frac{1}{0}$.

Page 172.

12. $2\frac{5}{1}\frac{5}{6}$.
13. $7.03\frac{1}{8}$.

Page 173.

3. $\$1\frac{1}{8}$.
4. $\$6\frac{1}{4}$.
5. $4\frac{1}{8}$ bbl.
6. $\$\frac{5}{21}$.
7. $\$1\frac{1}{4}$.
8. $\$4\frac{1}{5}$.
9. $\$\frac{11}{30}$.
10. $3\frac{1}{3}$ mi.

Page 174.

3. 1. $\frac{9}{15}$.
2. $\frac{8}{28}$.
3. $\frac{7}{50}$.
4. $\frac{7}{57}$.
5. $\frac{1}{17}$.
6. $\frac{2}{75}$.
7. $\frac{3}{97}$.
8. $\frac{19}{546}$.
9. $\frac{7}{205}$.
10. $\frac{1}{48}$.
11. $\frac{25}{1331}$.
12. $\frac{18}{281}$.
13. $\frac{14}{161}$.
14. $\frac{14}{231}$.
15. $\frac{2}{35}$.
16. $\frac{5}{98}$.
17. $\frac{11}{117}$.
18. $\frac{44}{1155}$.
19. $\frac{25}{3131}$.
20. $\frac{4}{45}$.
21. $4\frac{5}{18}$.
22. $7\frac{7}{24}$.
23. $2\frac{47}{126}$.
24. $2\frac{1}{4}$.
25. $2\frac{4}{5}$.
26. $12\frac{4}{9}$.
27. $58\frac{16}{33}$.
28. $37\frac{4}{9}$.
29. $20\frac{22}{41}$.
30. $1\frac{833}{304}$.

1. $\frac{8}{100}$.
2. $\frac{1}{5}$.
3. $3\frac{3}{4}$.
4. $7\frac{1}{2}$.
5. $95\frac{28}{50}$.
6. $224\frac{8}{9}$.
7. $\frac{11}{50}$.
8. $85\frac{4}{5}$.

Page 175.

9. $\$24\frac{24}{25}$.
10. $\frac{4}{5}$.
11. $5\frac{7}{12}$.
12. $\$44\frac{211}{315}$.
13. $\$1.37\frac{3}{5}$.
14. $\frac{1}{27}$.
15. $\frac{7}{12}$.
16. $11\frac{37}{40}$.
17. $5\frac{1}{2}$.
18. $4\frac{11}{45}$.

Page 177.

2. 1. $\frac{3}{8}$.
2. $\frac{15}{32}$.
3. $1\frac{7}{9}$.
4. $1\frac{5}{9}$.
5. $1\frac{5}{11}$.
6. $\frac{25}{144}$.
7. $\frac{15}{44}$.
8. $\frac{35}{36}$.
9. $\frac{45}{48}$.
10. $1\frac{13}{32}$.
11. $\frac{209}{217}$.
12. $\frac{44}{87}$.
13. $\frac{91}{100}$.
14. $\frac{23}{24}$.
15. $1\frac{2}{35}$.
16. $\frac{125}{216}$.
17. 10.
18. $2\frac{2}{45}$.
19. $2\frac{57}{75}$.
20. $19\frac{11}{70}$.
21. $8\frac{89}{311}$.
22. $\frac{6}{35}$.
23. $\frac{28}{33}$.
24. $\frac{671}{976}$.

3. $7\frac{17}{19}$.

5. 1. $1\frac{1}{3}$.
2. $\frac{8}{15}$.

Page 178.

3. $\frac{72}{85}$.
4. $\frac{1755}{4096}$.
5. $1\frac{44}{75}$.
6. $\frac{377}{512}$.
7. $14\frac{247}{432}$.
8. $\frac{24}{1113}$.
9. $\frac{15}{224}$.
10. $2\frac{4}{35}$.
11. $2\frac{1}{4}$.
12. 2.
13. $40\frac{3445}{30062}$.
14. $\frac{27}{246442}$.
15. 1.
16. $\frac{4}{5}$.

6. 1. $\frac{30}{133}$.
2. $2\frac{104}{135}$.
3. $\frac{11}{15}$.
4. $2\frac{1}{15}$.
5. $\frac{208}{385}$.
6. $8\frac{247}{276}$.
7. $\frac{5}{119}$.
8. $\frac{26}{171}$.
9. $\frac{443}{3430}$.
10. $\frac{35}{528}$.
11. $1\frac{81}{82}$.
12. $\frac{2}{45}$.
13. $\frac{650}{2277}$.
14. $6\frac{18}{37}$.
15. $\frac{35}{102}$.
16. $2\frac{5578}{9016}$.

Page 179.

17. 3.
18. 8.
19. $\frac{1}{8}$.
20. $1\frac{1}{16}$.

1. 16.
2. $5\frac{1}{2}$.
3. $18\frac{3}{4}$.
4. 15.
5. $11\frac{13}{17}$.
6. $293\frac{1}{3}$.
7. $6\frac{23}{48}$.
8. 10.
9. 76.
10. 52.
11. $6\frac{3}{4}$.
12. $7\frac{1}{2}$.

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13. $5\frac{3}{4}$.
14. $6\frac{11}{13}$.
15. $\frac{6}{5}$.
16. $6\frac{78}{360}$.
17. $880\frac{16}{21}$.

Page 181.

3. 1. $1\frac{11}{21}$.
2. 12.
3. $6\frac{11}{12}$.
4. $\frac{9}{20}$.
5. $\frac{35}{48}$.
6. $\frac{28}{33}$.
7. 20.
8. $\frac{25}{242}$.
9. $\frac{9}{11}$.
10. $\frac{153}{154}$.
11. 1.
12. $\frac{1}{3}$.
13. $2\frac{3}{5}$.
14. $1\frac{223}{425}$.
15. $\frac{115}{176}$.
16. $\frac{10}{11}$.
17. $2\frac{6}{35}$.
18. $\frac{1043}{1155}$.

Page 182.

1. 56.
2. 200.
3. 147.50.
4. $134\frac{13}{17}$.

Page 183.

5. 3920.
6. 2700.
7. 40.
8. 6750.
9. 144.

10 $6.60; $52.80.
11. $1622¼.
12. $100; $46⅔.
13. $4.90.
14. 6⅝.
15. $6400; $64000.
16. 4½ da.
17. $11.25.
18. $90.
19. 140; 120.

Page 184.

20. $10,125. 21. $\frac{5}{12}$.

Page 185.

1. $9\frac{89}{110}$.
2. $26\frac{21}{40}$.
3. $1\frac{17}{40}$.
4. $125.
5. $840.
6. 34⅜ cts.
7. 9 yd.
8. 54.
9. $1518¾.
10. $60.
11. $161.
12. $\frac{55}{102}$.

Page 186.

13. $4\frac{97}{112}$.
14. $19⅓.
15. $159\frac{71}{144}$.
16. 240.
17. 302⅔.
18. $\frac{9}{20}$.
19. 2⅔ da.
20. 372⅔; 497⅓.
21. $4\frac{88}{221}$.
22. 25.
23. $27½.
24. $57¾; $41⅓.

Page 187.

25. $2\frac{11}{12}$.
26. 176.
27. ⅓.
28. 11¼.
29. $4\frac{17}{40}$.
30. 160 A.
31. $3000.
32. 502½ da.
33. $\frac{2172}{2265}$.
34. $\frac{875}{1603}$.
35. Increased.
36. 80.
37. $10\frac{17}{60}$.
38. $726\frac{78}{420}$.
39. 28⅛ ft.
40. $\frac{726}{18585}$.
41. $\frac{1001}{1300}$.
42. 2520.
43. G. C. D., 2.

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44. $311.66⅔.
45. Horse, $140; cow, $40.
46. $78,545\frac{5}{11}$.
47. $\frac{825}{1284}$.
48. A. 15 da., B. 30 da.
49. $\frac{9}{20}$.
50. MLV.
51 1. $23\frac{851}{1320}$.
2. $\frac{28}{55}$.
3. $\frac{39}{77}$.
4. $3\frac{177}{350}$.
5. $28\frac{7}{16}$.
6. $\frac{851}{1320} \times \frac{1}{181}$.
7. 3345.
8. $\frac{206855}{302848}$.
9. $\frac{184}{135}$.
10. $\frac{5}{20}$.

Page 195.

2. 1. $\frac{9}{25}$.
2. $\frac{3}{4}$.
3. $\frac{9}{20}$.
4. $\frac{1}{2}$.
5. $\frac{13}{20}$.
6. $\frac{1}{2}$.
7. $\frac{3}{8}$.
8. $\frac{5}{8}$.
9. $\frac{7}{8}$.
10. $\frac{1}{8}$.
11. $\frac{1}{16}$.
12. $\frac{9}{50}$.
13. $\frac{3}{40}$.
14. $\frac{1}{20}$.
15. $\frac{7}{8}$.
16. $\frac{1}{200}$.
17. $\frac{9}{160}$.
18. $\frac{47043}{100000}$.
19. $\frac{8453}{31250}$.
20. $\frac{47}{1000000}$.
21. $\frac{3}{400}$.
22. $\frac{14}{125}$.
23. $\frac{81}{200}$.
24. $\frac{2}{625}$.
25. $\frac{3}{625}$.
26. $\frac{141}{400}$.
27. $\frac{27}{2500}$.
28. $\frac{1}{5000}$.
29. $\frac{3}{5000}$.
30. $\frac{7}{5000}$.
31. $\frac{29}{500}$.
32. $\frac{29}{400}$.
33. $\frac{123}{2000}$.
34. $\frac{281}{5000}$.
35. $\frac{638}{1250}$.
36. $\frac{7}{12}$.
37. $\frac{5}{6}$.
38. $\frac{13}{1500}$.
39. $\frac{7}{12}$.
40. $\frac{3}{800}$.

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3. 1. .3125.
2. .561.
3. .625.
4. .875.
5. .104.
6. 1.15625.
7. .3125.
8. .905.

9. .3167 +.
10. .2968 +.
11. .04.
12. .32.
13. .00378 +.
14. .03317 +.
15. .0244 nearly.
16. .012345 +.
17. .8338 +.
18. .0125.
19. .52.
20. .81 nearly.
21. .96$\frac{3}{7}$.
22. .58$\frac{12}{33}$.
23. .199$\frac{7}{33}$.
24. .34442 +.
25. .5555.
26. .30303.
27. .0192.
28. 10.0312 +.
29. .024.
30. .1066.
31. 12.3125.
32. 17.1875.
33. 14.8765 +.
34. 515.1466.
35. 231.032.

Page 198.

2. 1. 54.417.
2. 46.2946.
3. 51.543.
4. 64.894.
5. 34.1039.
6. 74 69.
7. 76.1575.
8. 45.3616.
9. 92.675.
10. 503.3016.
11. 426.2440.
12. 442.24.
13. 362.7795.
14. 103.976.

3. 1. 5707.7523.
2. 1087.7327.
3. 1889.4790.
4. 1198.3745.
5. 20479.1455.
6. 5377.4150.
7. 1017.3089.
8. 6573.1850.
9. 9401.2243.
10. 2158.0230.
11. 47.83929.

1. 35.703.
2. 897.0206163.
3. $2784.19.

Page 199.

4. 341.8674.
5. 513.13997.

Page 200.

2. 1. 11.980.
2. 11.955.
3. 14.914.
4. 16.944.
5. 19.57.
6. 23.175.
7. 30.7433.
8. 19.646.
9. 5.313.
10. 2.9904.
11. 5.3453.
12. 4.3353.
13. .684.
14. .027974.
15. .0624.
16. .372.67.
17. 864.005.
18. 293.988.
19. 6025.81.
20. 27.0417.

3. 1. 2.4216.
2. 3.9027.
3. 1.39444.
4. 2.7557.
5. 2.54975.
6. 1858.06.
7. 694.1589.
8. 6310.035.
9. 8814.4057.
10. 1983.013.

1. 4.725 yd.
2. .2312 in.
3. 3.76 mi.

Page 201.

4. $187.86.
5. 57.9.
6. $158.25.
7. $534.73.
8. 1.59964; .00236.
9. 59.0525.
10. $2.75.

Page 202.

3. 1. .2264.
2. 1.855.
3. 26.13.
4. .33698.
5. .05301.
6. .436475.
7. 3.50064.
8. 2.5098.
9. 1.4760.
10. .029078.
11. 1.094310.
12. 171.558.
13. 1.0044.
14. .0041335.
15. 2.45409.
16. .00172462.
17. .001287104.
18. .0657225.
19. .115589.
20. .1243920.
21. 145.41393.
22. 1015.075.
23. .215536.
24 17.653075.
25. 609.089.
26. 394.9528.
27. 3.1783.
28. 32175.5.
29. .232275.
30. .00000175.

Page 203.

1. $375.
2. $38.90.
3. $253.69.
4. $2.065.
5. $1842.00.
6. $201.91.
7. $1030.
8. $400.
9. $160.
10. $3.045.
11. $28.
12. $210.
13. $90.675.
14. $3339.00.
15. $5.625.
16. 7276.5 cu. in.
17. $3.75.
18. .29652.

Page 205.

4. 1. 6.4.
2. .315
3. .66.
4. .36.
5. 43.2.
6. 64.19 —.
7. 3650.
8. 365.
9. .005.
10. 4.78.

Page 206.

11. .0186956 +.
12. 1.3.
13. .02.
14. .3333.
15. 3000.
16. 2.5.
17. 36.4.
18. 43.2.
19. 82160.
20. 12345.
21. .32.
22. 960.
23. $16.68\frac{4}{15}$.
24. .204.
25. .0625.
26. 250.
27. 3.27.
28. 5.16.
29. 1.3.
30. 88.90.
31. 5.27.
32. 2.65.
33. .342.
34. $83.33\frac{1}{3}$.
35. $1666\frac{2}{3}$.
36. .000268.
37. 29176.
38. 1125.
39. 356.
40. 21.24.
41. .1257.
42. 10,000,000,000.
43. 1.001.
44. 426.
45. 5.266 +.
46. 7901.2.

Page 208.

1. $54.75.
2. 182 A.

Page 209.

3. 28 bu.
4. 96 doz.
5. $5\frac{1}{4}$ yd.
6. 9 lb.
7. 188 T.
8. 50.385 + T.
9. 101 bbl.
10. 28.8 bu.
11. 8.
12. $1.259.
13. 12 cd.
14. 18.
15. 1. $.50.
2. 4 cts.
3. 90 cts.
4. $21.33\frac{1}{3}$.
5. $.87\frac{1}{2}$.

Page 210.

6. $.05.
7. $.175.
8. $.06\frac{1}{4}$.
9. $.08.
10. $5.35.
16. 1. .48.
2. .00301.
3. 72.63.
4. 1.70.
5. 6.03.
6. 27.
7. 1.
8. .06.
9. 69.917.
10. 646.

Page 214.

1. $2.25.
2. $778.615.
3. $63.41.
4. $876.24.
5. $1036.351.
6. $291.84.
7. $52.827.
8. $.15.
9. $28.
10. $14.875.
11. $16.50.

Page 215.

12. $1.65.
13. $19.12½.
14. $4234.37½.
15. 1½ lb.
16. 28 yd.
17. 96 lb.
18. $182.82.
19. 18 wk.
20. 40 wk.
21. 42 qt.
22. $3.40.
23. $148.43¾.
24. 44⅘ mo.
25. $637.50.
26. $13.485.

Page 216.

27. 6½ cts.
28. 12 doz.
29. $7.71¼.
30. 1864 lb.
31. $294⅜.
32. ½ yr.
33. $360.
34. $1081.25.

Page 217.

1. $9.415.

Page 218.

2. $269.435.
3. $1866.45.
4. $2610.97.

Page 219.

6. $65.57.
7. $48.89.
8. $21.86.
9. $164.51.

Page 220.

10. $6917.55.
11. $17.89.

Page 231.

3. 8 pk.
4. 320 sq. rd.
5. 180 in.
6. 24 gal.
7. 200 min.
8. 100 qr.

Page 232.

9. 96 oz.
10. 20 pk.
11. 24 pt.
12. 140 pwt.
13. 3 sq. yd.
14. 20 wk.
15. 10 doz.
16. 144 shts.
17. 3 cu. yd.
18. 2592 cu. in.
19. 6 pwt.
20. 6 lb.
21. 100 units.
22. 24 doz.
23. 72 doz.
24. 800 cts.
25. 75 cts.
26. $1.00.

Page 233.

2. 1. 225 in.
2. 500 in.
3. 611 in.
4. 1402 in.
5. 1783 in.
6. 63 pt.
7. 117 pt.
8. 215 pt.
9. 239 pt.
10. 467 pt.
11. 19,586 sec.
12. 77,009 sec.
13. 106,246 sec.
14. 146,440 sec.
15. 8665 oz.
16. 15,789 oz.
17. 107,053 oz.
18. 173,546 oz.
19. 336,170 oz.
20. 54,509 gr.

3. 1. 37,858 gr.
2. 68,915 gr.
3. 333 pt.
4. 1786 gi.
5. 491 pt.
6. 6463 pt.
7. 1165 in.
8. 1322 in.
9. 1889 in.
10. 3613 in.
11. 5923 in.
12. 8883 sq. in.
13. 13,965 sq. in.
14. 18,156 sq. yd.
15. 44,674 sq. yd.
16. 89,328 sq. yd.
17. 159,072 cu. in.
18. 246,201 cu. in.
19. 483,850 cu. in.

Page 234.

20. 1262 ℈.
21. 1675 ℈.
22. 3143 sc.
23. 2203 shts.
24. 2637 shts.
25. 1111 pwt.
26. 1996 sc.
27. 10,776 min.
28. 1997 in.
29. 33,636¾ sq. rd.
30. 548,658 in.

Page 235.

1. 2 yd. 0 ft. 2¼ in.
2. 3 da. 21 hr. 20 min.
3. 1 ft. 8⅞ in.
4. 2 pk. 6.4 qt.
5. 11 cwt. 20 lb.
6. 13 cu. ft. 864 cu. in.
7. 53 rd. 1 yd. 2 ft. 6 in.
8. 91 sq. rd. 12 sq. yd. 8 sq. ft. 97⅞ sq. in.
9. 5 da. 21 hr. 7 min. 12 sec.
10. 1 qt. 1 pt. 1.5552 gi.
11. 1 ft. 9.06 in.
12. 12 R. 10 qr.
13. 9 Gr. 4 doz.
14. 6ℨ 5ʒ 1℈.
15. 8 oz. 8 pwt.
16. 17 cwt. 77 lb. 14 oz.
17. 2 pk. 5 qt. 0 pt. 2¾ gi.
18. 3 qt. 1 pt.
19. 2 cd. ft. 12.2368 cu. ft.
20. 19 sq. rd. 22 sq. yd. 4.554 sq. ft.
21. 293 rd. 1 yd. 2 ft. 6 in.
22. 240 A.
23. 11 cu. ft. 432 cu. in.
24. 3 qt. 1 pt.
25. 1 pk. 4 qt. 1.6 pt.
26. 21½ gr.
27. 5.76 gr.
28. 25.55 da.
29. 18.018 units.
30. 16 qr. 16 sheets.

Page 236.

3. 1. 166 gal. 1 qt. 1 pt.
   2. 537 gal.
   3. 213 gal. 2 qt. 3 gi.
   4. 23 rd. 2 yd. 1 ft. 2 in.
   5. 18 rd. 3 in.
   6. 13 rd. 3 yd. 2 ft.
   7. 73 bu. 1 qt. 1 pt.
   8. 90 bu. 2 pk. 1 qt.
   9. 63 bu. 1 pk. 5 qt.
   10. 3 cwt. 92 lb. 3 oz.
   11. 5 cwt. 77 lb. 6 oz.
   12. 3 T. 94 lb.
   13. 2 T. 8 cwt. 6 lb.
   14. 32 wk. 20 hr.
   15. 2 hr. 34 min. 39 sec.
   16. 7 da. 13 hr. 45 min.
4. 1. 3 sq. yd. 1 sq. ft. 22 sq. in.
   2. 4 sq. yd. 6 sq. ft.
   3. 6 R. 1 qr. 1 sheet.
   4. 7 R. 2 qr. 18 sheets.
   5. 5 cu. ft. 335 cu. in.
   6. 1 lb. 4 oz. 7 pwt. 17 gr.

Page 237.

   7. 1 lb. 7 oz. 15 pwt. 17 gr.
   8. 1 lb. 9ℨ 2ʒ 2℈ 9 gr.
   9. 1 C.
   10. 9 A.

6. 1. $\frac{7}{8}$ yd.
   2. $\frac{9}{16}$ pk.
   3. $\frac{11}{8}$ bu.
   4. $\frac{31}{32}$ gal.
   5. $\frac{52}{64}$ bu.
   6. $\frac{211}{128}$ rd.
   7. $\frac{3}{4}$ wk.
   8. $\frac{1041}{4000}$ da.
   9. $\frac{223}{5000}$ T.
   10. $\frac{11}{64}$ bu.
   11. $\frac{15}{112}$ bbl.
   12. $\frac{201}{400}$ E.
   13. $\frac{101}{192}$ lb.
   14. $\frac{451}{438}$ yr.

Page 238.

2. 1. 63 gal. 1 qt.
   2. 61 bu. 1 pk. 1 qt.
   3. 26 da. 12 hr. 52 sec.
   4. 214 lb. 3 oz.
   5. 68 lb. 0 oz. 19 pwt. 3 gr.
   6. 88 yd. 1 ft. 2 in.
   7. 128 gal. 1 qt. 0 pt. 3 gi.
   8. 98 A. 96 sq. rd.

Page 239.

9. 42 T. 14 cwt. 68 lb. 9 oz.
10. 26 yd. 2 ft. 5 in.

2. 4 bu. 3 pk. 7 qt.
3. 4 bu. 0 pk. 6 qt.
4. 6 gal. 3 qt. 1 pt. 3 gi.
5. 2 da. 22 min. 20 sec.
6. 32 lb. 10 oz. 3 pwt.
7. 20 yd. 1 ft. 9 in.
8. 14 rd. 0 ft. 0 in.
9. 15 lb. 10 oz.
10. 54 sq. rd. 264 sq. ft. 69 sq. in.

Page 240.

11. 14 T. 16 cwt. 39 lb. 14 oz.
12. 3 gal. 1 qt. 1 pt. 2 gi.

2. 1. 52 yr. 5 mo. 5 da.
   2. 15 yr. 6 mo. 8 da.
   3. 52 yr. 4 mo. 29 da.
   4. 10 yr. 10 mo. 17 da.
   5. 84 yr. 3 mo. 11 da.

Page 241.

2. 1. 45 bu. 2 pk. 0 qt.
   2. 47 gal. 0 qt. 0 pt. 1 gi.
   3. 59 lb. 8 oz. 2 pwt. 9 gr.
   4. 70 lb. 6 oz. 6 dr. 2 ℈ 8 gr.
   5. 1 da. 3 hr. 4 min. 36 sec.
   6. 29 T. 19 cwt. 38 lb. 14 oz.
   7. 38 rd. 5 yd. 2 ft. 2 in.
   8. 165 cu. yd. 8 cu. ft. 1670 cu. in.
   9. 56 sq. yd. 2 sq. ft. 48 sq. in.
   10. 31 da. 23 hr. 13 min.
   11. 216 rd. 3 yd. 2 ft. 3 in.
   12. 64 R. 17 qr. 2 sheets.
   13. 58 bbl. 30 gal.
   14. 32 yr. 3 mo. 18 da. 18 hr.
   15. 53 T. 3 cwt. 66 lb. 4 oz. 8 dr.

2. 1. 4 gal. 2 qt. 1 pt. $1\frac{1}{2}$ gi.
   2. 5 bu. 1 pk. 4 qt. 1 pt.
   3. 3 yd. 0 ft. $3\frac{7}{8}$ in.
   4. 5 cwt. 79 lb. $15\frac{1}{8}$ oz.

Page 242.

5. 5 lb. 4 oz. $11\frac{1}{2}$ pwt.
6. 3 lb. 8 oz. 7 dr. $2\frac{5}{8}$ ℈.
7. 2 hr. 53 min. $6\frac{5}{7}$ sec.
8. 6 sq. yd. 5 sq. ft. 40 sq. in.
9. 4 rd. 2 yd. 2 ft. $5\frac{1}{2}$ in.
10. 2 bbl. 1 gal. 1 qt. $1\frac{1}{2}$ pt.
11. 20 bu. 2 pk. $5\frac{1}{7}$ qt.
12. 3 gal. 2 qt. 0 pt. $1\frac{1}{11}$ gi.
13. 1 mi. 183 rd. 4 ft. $8\frac{8}{11}$ in.
14. 5 A. 46 sq. rd. $21\frac{1}{2}$ sq. yd.
15. 1 T. 15 cwt. 11 lb. $8\frac{1}{7}$ oz.

Page 243.

2. 432 sq. ft.
3. 300 sq. ft.
4. 204 sq. ft.; $22\frac{2}{3}$ sq. yd.
5. 144 sq. ft.; 16 sq. yd.
6. 912 sq. ft.; $101\frac{1}{3}$ sq. yd.

Page 244.

7. 19,500 sq. rd.; $121\frac{7}{8}$ A.
8. 750 sq. ft.; $10.42.
9. $32.30.
10. 36 A. 16 sq. rd.
11. $2025.
12. $48.
13. 110.274 A.
14. $84.
15. $99,975.
16. 49 sq. rd.
17. $11\frac{47}{160}$ A.
18. $50.65.
19. 10 A.
20. $36.

Page 246.

2. 1. 1120 cu. ft.
2. 1080 cu. in.
3. 540 cu. yd.
4. 1650 cu. ft.
5. 665 cu. ft.
6. 1200 cu. ft.
7. 1668⅓ cu. ft.
8. 1431 cu. ft.
9. 840¾ cu. ft.
10. 64,375 cu. ft.

Page 247.

3. 1. $26.25.
2. $69.88.
3. $566.26.
4. $864.00.
5. $5871.09.

4. 1. $72.73.
2. $170.45.
3. $135.07.
4. $5818.18.
5. $15.91.

5. 1. $71.11.
2. $33.33.
3. $152.00.
4. $360.19.
5. $152.21.

Page 248.

2. 1. 10⅔ ft.
2. 11¼ ft.
3. 10 ft.
4. 10 ft.
5. 21 ft.
6. 90 ft.
7. 333⅓ ft.
8. 216 ft.
9. 33⅓ ft.
10. 6 ft.

3. $3.24.
4. $72.
5. 90$\frac{1}{64}$ lb.
6. $15.00.

Page 249.

7. 402½ ft.

Page 251.

8. 1. $6.
2. $14.
3. $24.
4. $40.
5. $62.50.
6. 77 sheep.
7. 1971 bu.
8. 1250 bbl.
9. 18 men.
10. 20 horses.
11. $.39.
12. $.6265.
13. $.848.
14. $.525.
15. $3.0072.
16. 56¼ lb.
17. 1414.92 lb.
18. 7260 apples.
19. 4440 dolls.
20. 632⅔ yr.

4. $53.40.
5. $40.25.
6. $250; $281.25.
7. 10 cts.
8. $1363.95.
9. 56 quarts.
10. 6 pupils.
11. 2 words.

Page 252.

12. $840.
13. $296.10.
14. $487.50.
15. $1915.

Page 253.

2. 1. $15.
2. $15.
3. $20.
4. $18.
5. $70.
6. $44.
7. $35.40.
8. $401.52.
9. $150.80.
10. $1366.08.
11. $1.68.
12. $4.42.
13. $17.52.
14. $13.08.
15. $12.49.
16. $29.17.
17. $27.35.
18. $37.87.
19. $874.28.
20. $21.66.

Page 254.

1. $98.12.
2. $137.79.
3. $177.00.
4. $297.43.
5. $308.41.
6. $90.00.
7. $227.35.
8. $141.29.
9. $176.08.
10. $123.36.

Page 255.

1. $104.89.
2. $164.20.
3. $57.50.
4. $180.51.
5. $234.83.
6. $141.00.
7. $79.352.
8. $77.98.
9. $116.12.
10. $68.67.

GENERAL REVIEW.

1. 1. $776,475.2675.
2. $899,721.54.
3. $877,968.74.
2. 828.023.
3. 901.3006.

4. 52 da. 12 hr. 12 min.
5. 106 bu. 1 pk.
6. 73 lb. 4 oz.
7. 25 rd. 5 yd. $0\frac{1}{2}$ ft.
8. 224.82.
9. $7\frac{12}{133}$.
10. 653 cu. yd. 8 cu. ft. 1124 cu. in.
11. (1) 1979323; (2) 948009; (3) 1439615.
12. (1) $2\frac{7}{12}$; (2) $5\frac{14}{15}$; (3) $3\frac{21}{24}$; (4) $6\frac{10}{11}$; (5) 482.87; (6) 2120.35; (7) 88.953; (8) 4240.7.
13. 484.006651.
14. 44 yr. 4 mo. 12 da.
15. 11 rd. 4 yd. 6 in.
16. 1 bu. 0 pk. 7 qt.
17. 5 gal. 0 qt. 1 pt.
18. 4.1722.
19. 17.
20. 1689.375.
21. (1) 119173214; (2) 229382875; (3) 149155212.
22. $40\frac{1}{8}$; $28\frac{124}{171}$; $66\frac{11}{12}$; $6\frac{101}{176}$.
23. 5.81096; 73.08225; .03396.
24. $14.50.
25. $1944.39.
26. 167 mi. 165 rd. $5\frac{1}{2}$ ft.
27. $46.67.
28. $\frac{88}{80}$.
29. $\frac{528}{2520}$.
30. 92 hhd. 57 gal. 1 qt. 1 pt.
31. $12397\frac{1}{5005}$; $9790\frac{113}{4480}$.
32. $1\frac{1}{6}$; $19\frac{28}{77}$; $2\frac{81}{132}$; $\frac{55}{62}$.
33. 21; 12360; 43647.9.
34. 3 mi. 232 rd. $11\frac{1}{2}$ ft.; 8 cd. $30\frac{5}{8}$ ft.
35. $\frac{45}{1232}$.
36. $9\frac{85}{2772}$.
37. 80 bbl.
38. 35 da.
39. 200 C.
40. $\frac{7}{24}$.
41. $\frac{2}{35}$; 120; 28.
42. 75.
43. $\frac{7}{15}$; $\frac{441}{500}$; $\frac{4751}{5044}$; $\frac{5}{8}$.
44. 5, 59; 2, 2, 139; 2^3, 3^4; none; 5, 13, 23.
45. $\frac{15}{27}$; $\frac{377}{816}$; $\frac{700}{1000}$.
46. $33\frac{3}{7}$; $82\frac{8}{44}$; $62\frac{1}{2}$; $13\frac{11}{72}$; 444.
47. $\frac{479}{7}$; $\frac{26402}{91}$; $\frac{10716}{111}$; $\frac{25}{8}$.
48. 4196 gi.; 2580 pt.
49. 13 mi. 109 rd. 2 yd. 1 ft. 10 in.; 37037 cu. yd. 1 cu. ft.
50. 17 cwt. 77 lb. $12\frac{4}{5}$ oz.; 3 pk.; 518 A. 64 sq. rd.; 9 hr. 14 min. 24 sec.
51. $42\frac{5}{6}$.
52. 76.5466.
53. .3706883.
54. 87.45; 874.5; 87450; 874500.
55. .008745; .0008745; .000008745; .0000008745.
56. .04965+; 1000000000; .0833+.
57. 13 bbl.
58. $52.437.
59. 4 hr.
60. $2.25.
61. $39\frac{3}{8}$ A.
62. $751\frac{1}{3}$ cu. ft.
63. $15\frac{11}{32}$ cd.
64. 7,762,392 cu. in.
65. $1013\frac{1}{3}$ bd. ft.
66. 2.453 A.
67. 988 sq. ft.
68. $3246.99.
69. 63 bd. ft.
70. $22.28.
71. $209.44.
72. $.86.
73. $1.73.
74. $963.77.
75. $515.20.
76. $159.22.
77. $65.17; $565.17.
78. $5.00; $5.50.

79. $105.41.
80. $120.38.
81. 1463 mi.
82. A. 55, B. 54, C. 100.
83. 307.
84. 27,735.
85. 55,111.
86. 1584.
87. 850 bu.
88. $20,000; $12,500.
89. $5\frac{5}{11}$ da.
90. 157 A. 144 sq. rd.
91. $443.30.
92. $52.54.
93. 45.
94. 64 rd.
95. 249; 332.
96. $45.00.
97. $316 27.
98. $7\frac{8}{21}$.
99. $5.62\frac{1}{2}$.
100. B. $= \frac{P.}{R.}$; B. $= \frac{Int.}{R. \times T.}$; R $= \frac{P.}{B.}$; R. $= \frac{Int.}{B. \times Time.}$ Time $= \frac{Int.}{P.}$; T. $= \frac{Int.}{B. \times R.}$